Recent developments in statistics methods for HEP and their implications to Astroparticle physics

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Plan of the talk

- Basic definitions of the statistics of hypotheses test
- The profile likelihood and its asymptotic formulae
- The CLs and PCL methods
- Discovery of a Higgs Boson or a WIMP
- The look elsewhere effect
- Exclusion of a Higgs Boson or a WIMP

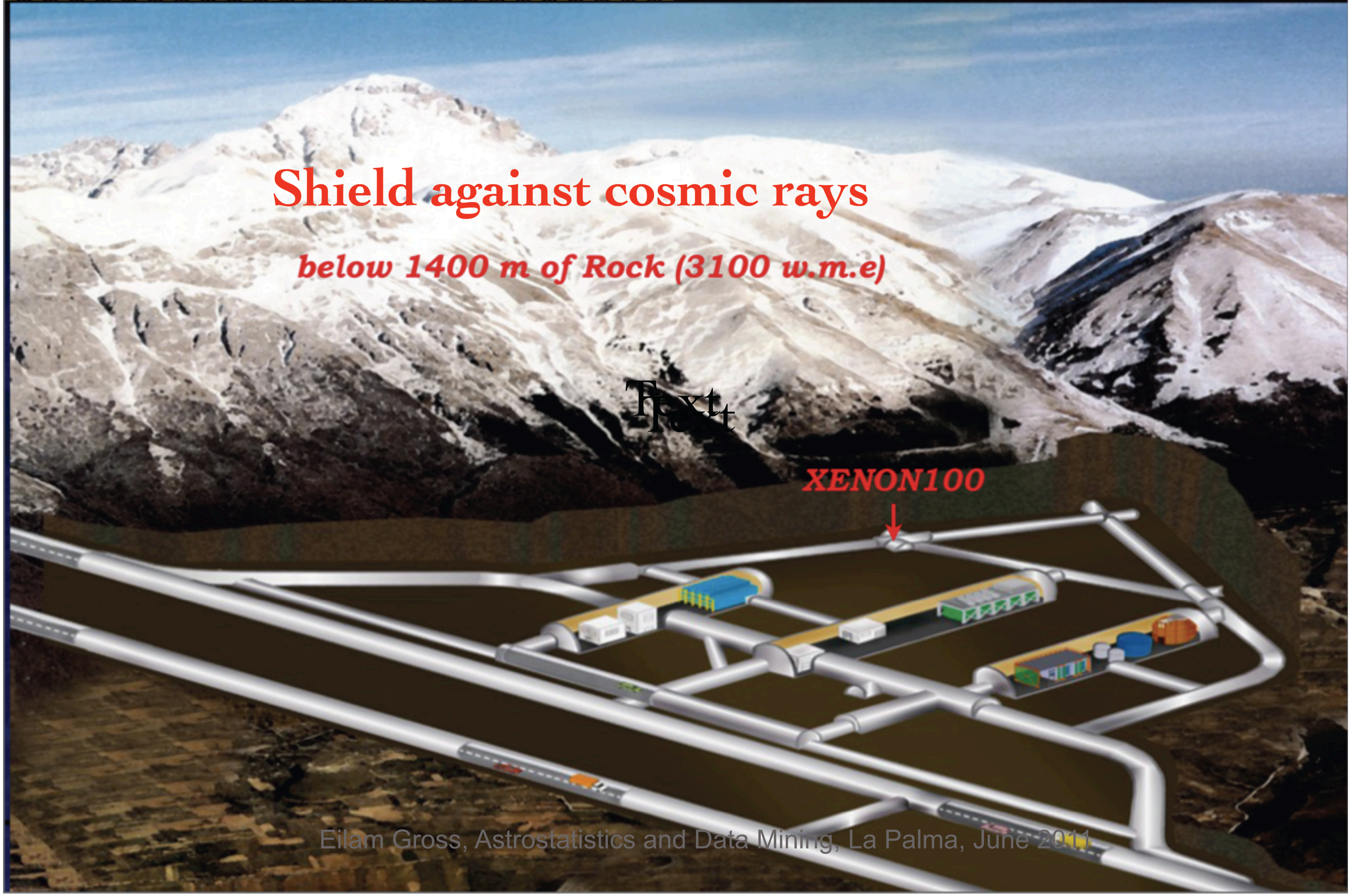
What is the statistical challenge in HEP?

- High Energy Physicists (HEP) have an hypothesis: The Standard Model.
- This model breaks unless there exists its only one ingredient, yet to be discovered: the Higgs Boson
- The minimal content of the Standard Model includes the Higgs Boson , but extensions of the Model include other particles which are yet to be discovered
- The challenge of HEP is to generate tons of data and to develop powerful analyses to tell if the data contains evidence for new particles, perhaps expected, but yet to be discovered

The Large Hadron Collider (LHC)

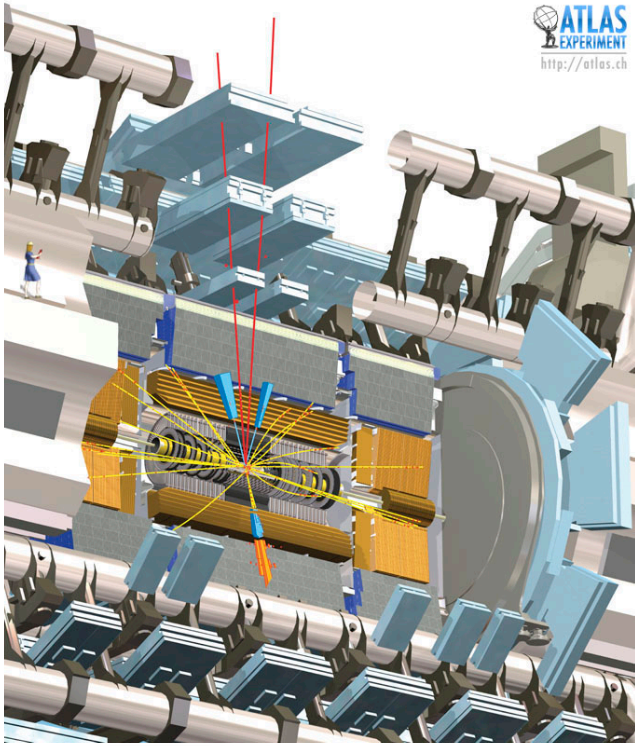
The LHC is a very powerful accelerator aims to produce 10^9 proton-proton collisions per sec aiming to hunt a Higgs with a 10^{-12} production probability

XENON100 @ Gran Sasso Underground Lab

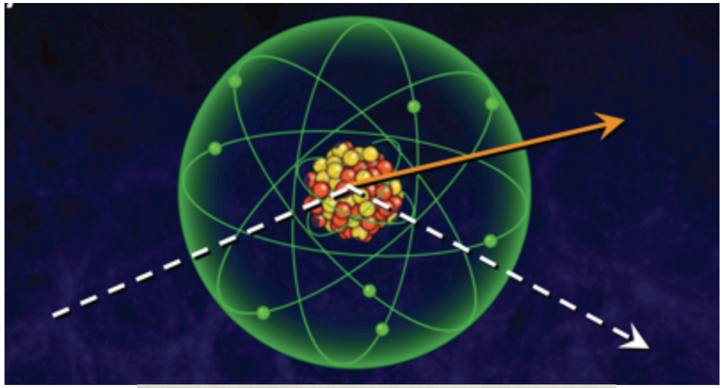


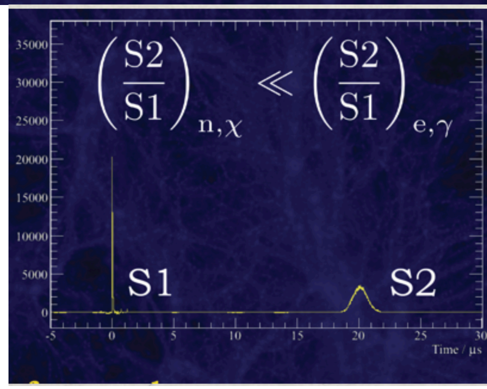
Dark Matter vs Higgs Search

- The common factor is : both are searches for rare events
- Higgs can be buried under ~10^{12} collision with $\sigma < 10^{-34}\ \mathrm{cm}^2$



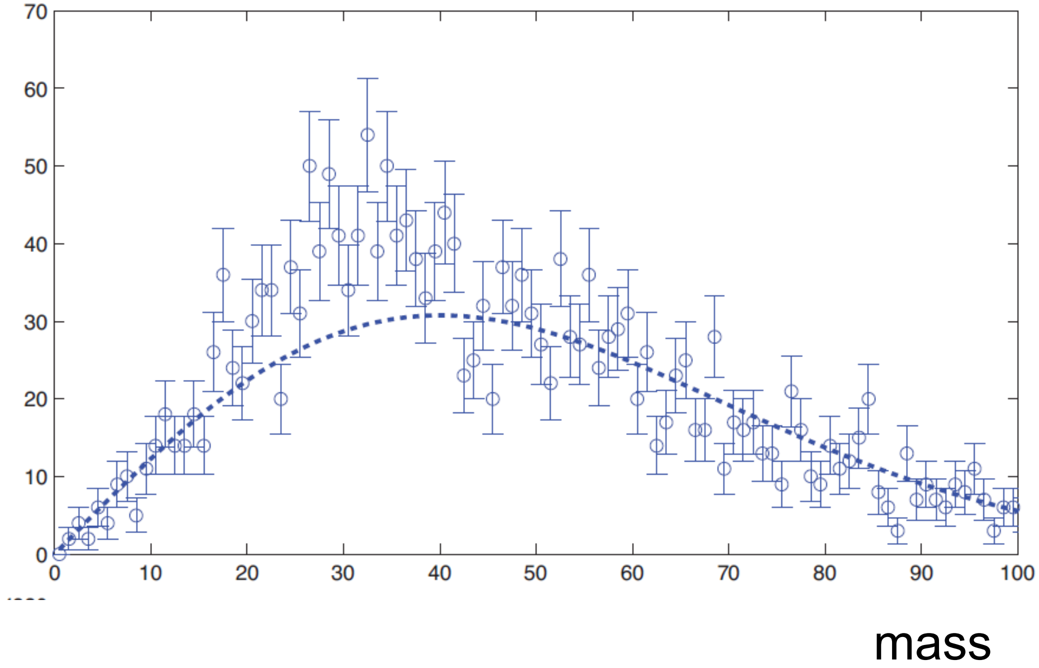
- Out of 10^{15} WIMPS that pass through your body every day, <10 interact!
- $\sigma < 10^{-44}\ \mathrm{cm}^2$





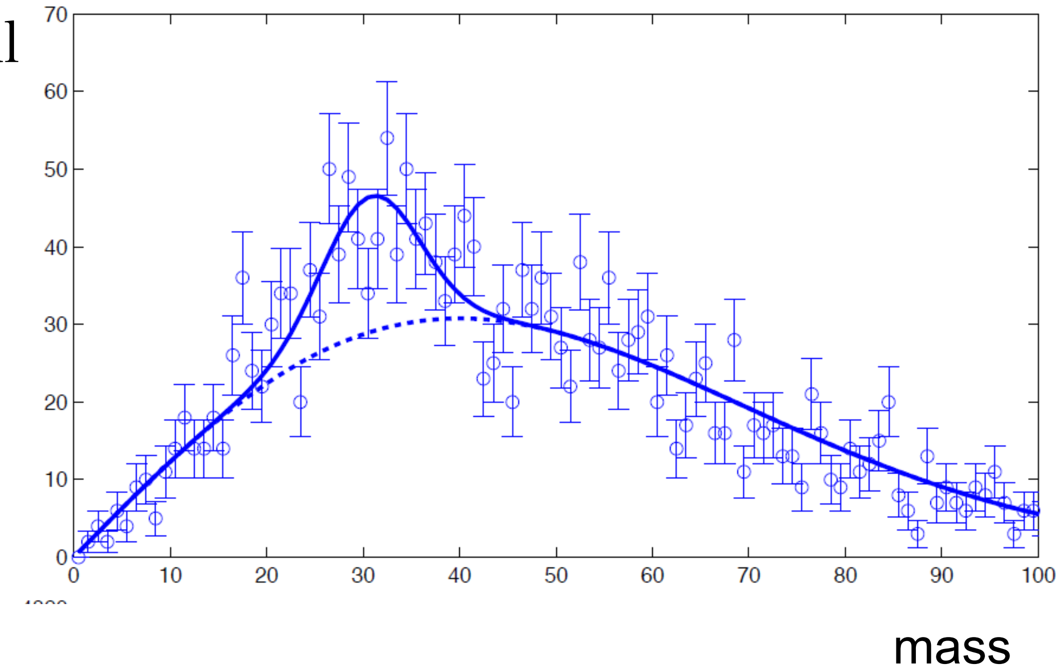
The Statistical Challenge of HEP

- The DATA: Billions of Proton-Proton collisions which could be visualized with histograms
- The Higgs mass is unknown
- In this TOY example, we ask if the expected background (the Standard Model WITHOUT the Higgs Boson) contains a Higgs Boson, which would manifest itself as a peak in the distribution



The Statistical Challenge of HEP

- So the statistical challenge is obvious:
- To tell in the most powerful way, and to the best of our current scientific knowledge, if there is new physics, beyond what is already known, in our data
- The complexity of the apparatus and the background physics suffer from large systematic errors that should be treated in an appropriate way.



- Is there a signal hidden in this data?
- What is its statistical significance?
- What is the most powerful test statistic that can tell the SM from an hypothesized signal?

The Null Hypothesis

- The Standard Model without the Higgs is an hypothesis, (BG only hypothesis), denoted by H_0
- In the absence of an alternate hypothesis, one would like to test the compatibility of the data with the given H_0 hypothesis.
- This is actually a **goodness of fit test**
- We are interested to tell between **two** hypotheses the BG-only hypothesis and the s+b hypothesis!

A Tale of Two Hypotheses

NULL | ALTERNATE

- Test the Null hypothesis and try to reject it
- Fail to reject it OR reject it in favor of the Alternate hypothesis

A Tale of Two Hypotheses

NULL

H_0- SM w/o Higgs

ALTERNATE

H_1- SM with Higgs

- Test the Null hypothesis and try to reject it
- Fail to reject it OR reject it in favor of the Alternate hypothesis

H_1- SM with Higgs

- Higgs with a specific mass m_H

  OR

- Higgs anywhere in a specific mass-range
  - The look elsewhere effect

A Tale of Two Hypotheses

NULL

H_0- SM w/o Higgs

ALTERNATE

H_1- SM with Higgs

- Test the Null hypothesis and try to reject it
- Fail to reject it OR reject it in favor of the Alternate hypothesis

- Reject H_0 in favor of H_1 – A DISCOVERY

A Tale of Two Hypotheses

NULL

H_1- SM with Higgs

ALTERNATE

H_0- SM w/o Higgs

- Test the Null hypothesis and try to reject it
- Fail to reject it OR reject it in favor of the Alternate hypothesis

- Reject H_0 in favor of H_1 – A DISCOVERY

- Reject H_1 in favor of H_0 – Excluding H_1

Testing an Hypothesis (wikipedia...)

- The first step in any hypothesis testing is to state the relevant **null,** H_0 and **alternative hypotheses,** say, H_1
- The next step is to define a test statistic, Q, under the null hypothesis
- Compute from the observations the observed value q_{obs} *of the test statistic* Q.
- Decide (based on q_{obs}) to either
  fail to reject the null hypothesis or
  reject it in favor of an alternative hypothesis

- **next: How to construct a test statistic, how to decide?**

DISCOVERY

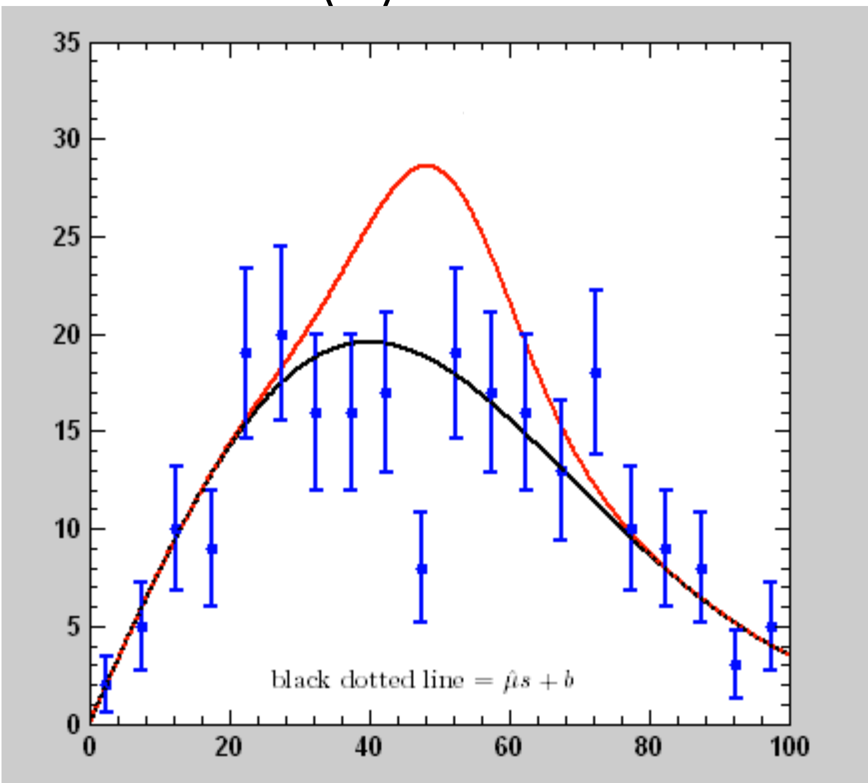
Test Statistic

- To construct a test statistic one needs a **model**
- $L(H_0) \sim Prob(data \mid H_0)$
- $L(H_1) \sim Prob(data \mid H_1)$
- Note: The Likelihood as indicated by its name, is the compatibility of a **given** data set with an hypothesis. If the data changes, so is the Likelihood!

The Toy Physics Model

- The NULL hypothesis H_0: SM without Higgs Background Only

$$\langle n \rangle = b$$

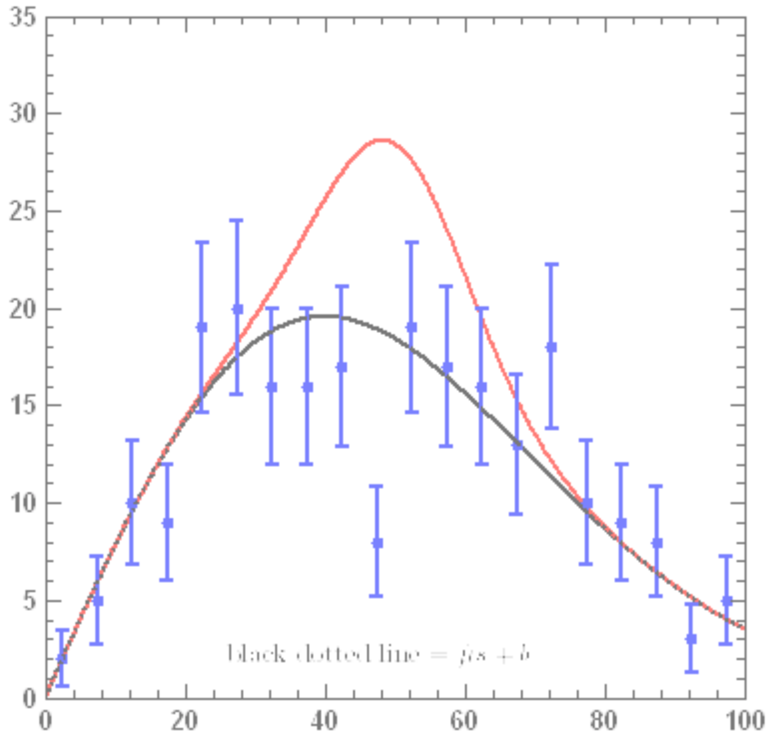


mass

The Toy Physics Model

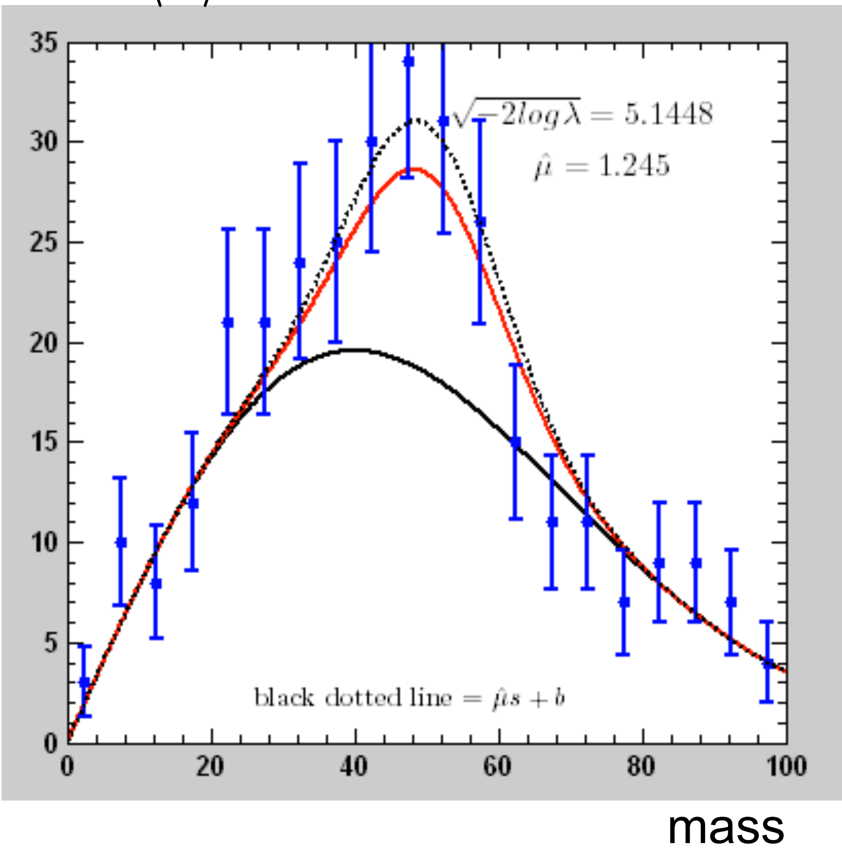
- The NULL hypothesis H_0: SM without Higgs Background Only

$$\langle n \rangle = b$$



- The alternate Hypothesis H_1:
- SM with a Higgs with a mass m_H

$$\langle n \rangle = s(m_H) + b$$

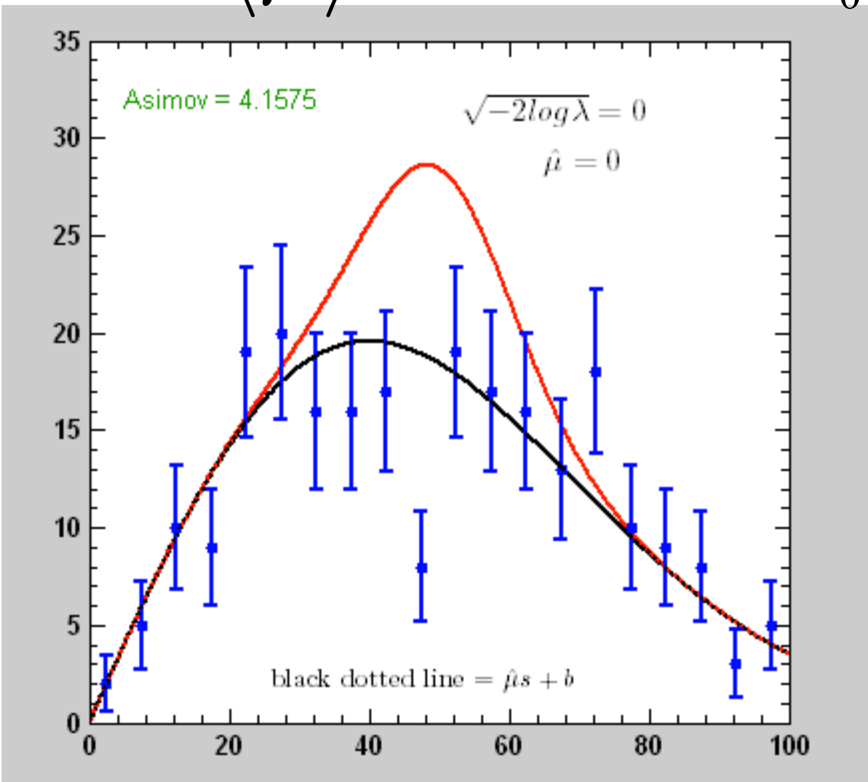


The Toy Physics Model

$$n = \mu s + b \qquad n = \mu s + b$$

$$MLE \;\; \hat{\mu} \qquad MLE \;\; \hat{\mu}$$

$$\langle \hat{\mu} \rangle = 0 \;\; under \;\; H_0 \qquad \langle \hat{\mu} \rangle = 1 \;\; under \;\; H_1$$



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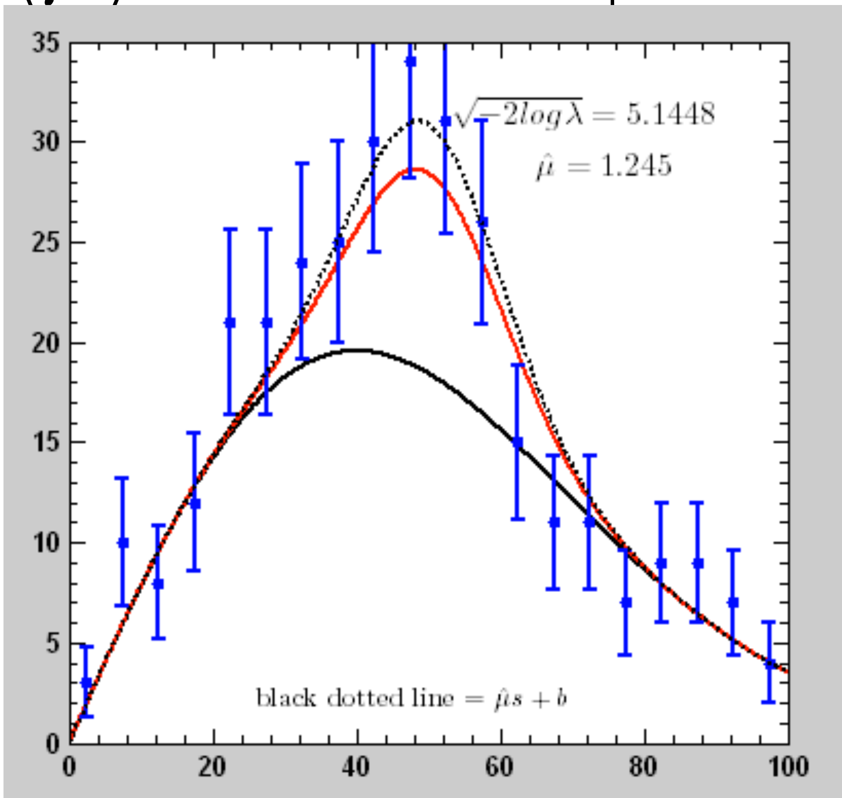


mass

The Profile Likelihood ("PL")

- For discovery we test the H_0 null hypothesis and try to reject it

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$

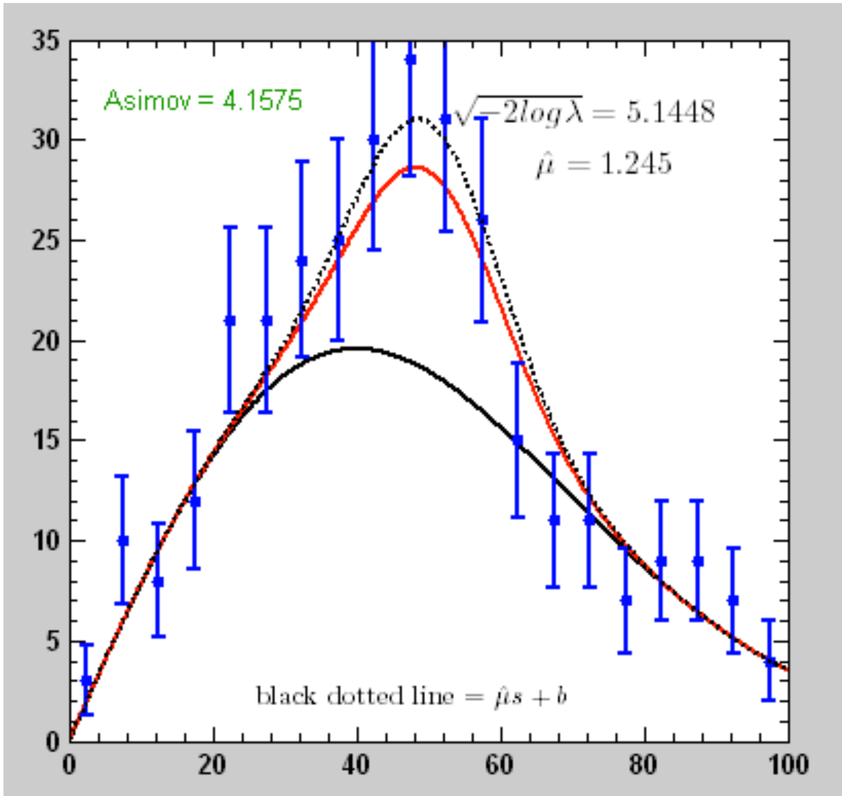
$$\hat{\mu} > 0$$

- For $\hat{\mu} \sim 0,\ q$ small

  $\hat{\mu} \sim 1, q$ large

- In general: testing the H_μ hypothesis i.e., a SM with a signal of strength μ,

$$q_\mu = -2\ln\frac{L(\mu)}{L(\hat{\mu})}$$



The PDF of the test statistic

- No, not the **P**arton **D**istribution **F**unction

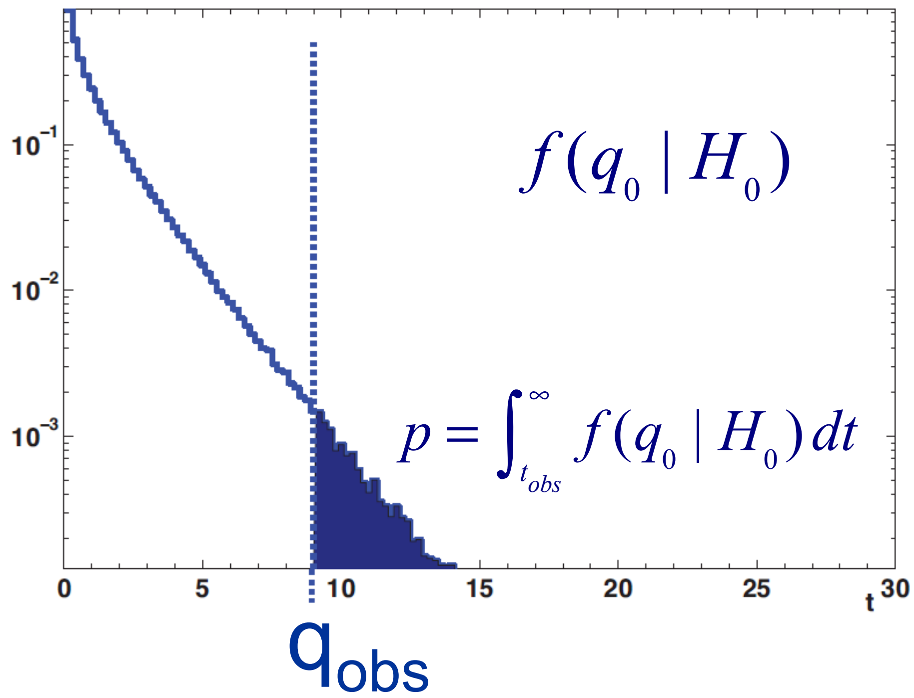
- Not a **P**ortable **D**ocument **F**ormat

- We need to know the **P**robability **D**istribution **F**unction of the test statistic under the null hypothesis $f(q_0 \mid H_0)$

$$q_0 = -2 \ln \frac{L(b)}{L(\hat{\mu} s + b)} \quad \hat{\mu} > 0$$

Significance & p-value

- Calculate the test statistic based on the observed experimental result (after taking tons of data), q_{obs}
- Calculate the probability that the observation is as or less compatible with the background only hypothesis (p-value)

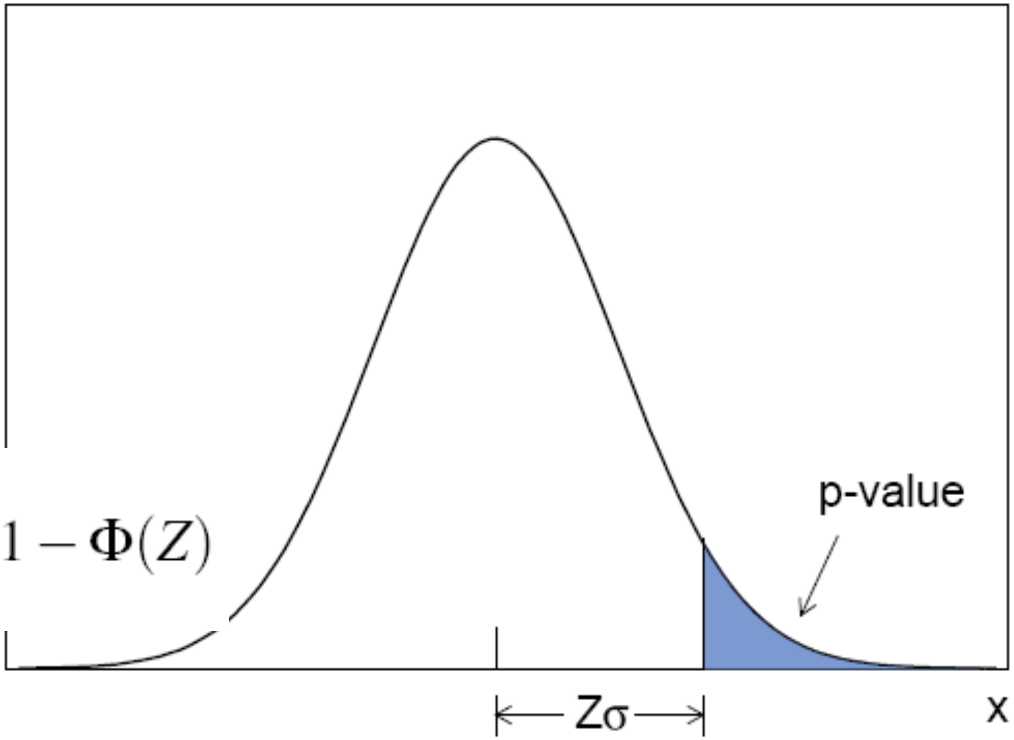


$$p = \int_{t_{obs}}^{\infty} f(q_0 | H_0)\, dq_0$$

If p-value< $2.8 \cdot 10^{-7}$, we claim a 5σ discovery
0.025 corresponds to 2σ
0.16 corresponds to 1σ

From p-values to Gaussian Significance

- It is a custom to express the p-value as the significance associated to it, had the PDF been Gaussians



$$p = \int_Z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = 1 - \Phi(Z)$$

$$Z = \Phi^{-1}(1-p)$$

A significance of $Z = 5$ corresponds to $p = 2.87 \times 10^{-7}$.

A significance of Z=1.64 corresponds to p=5%

The Profile Likelihood ("PL")

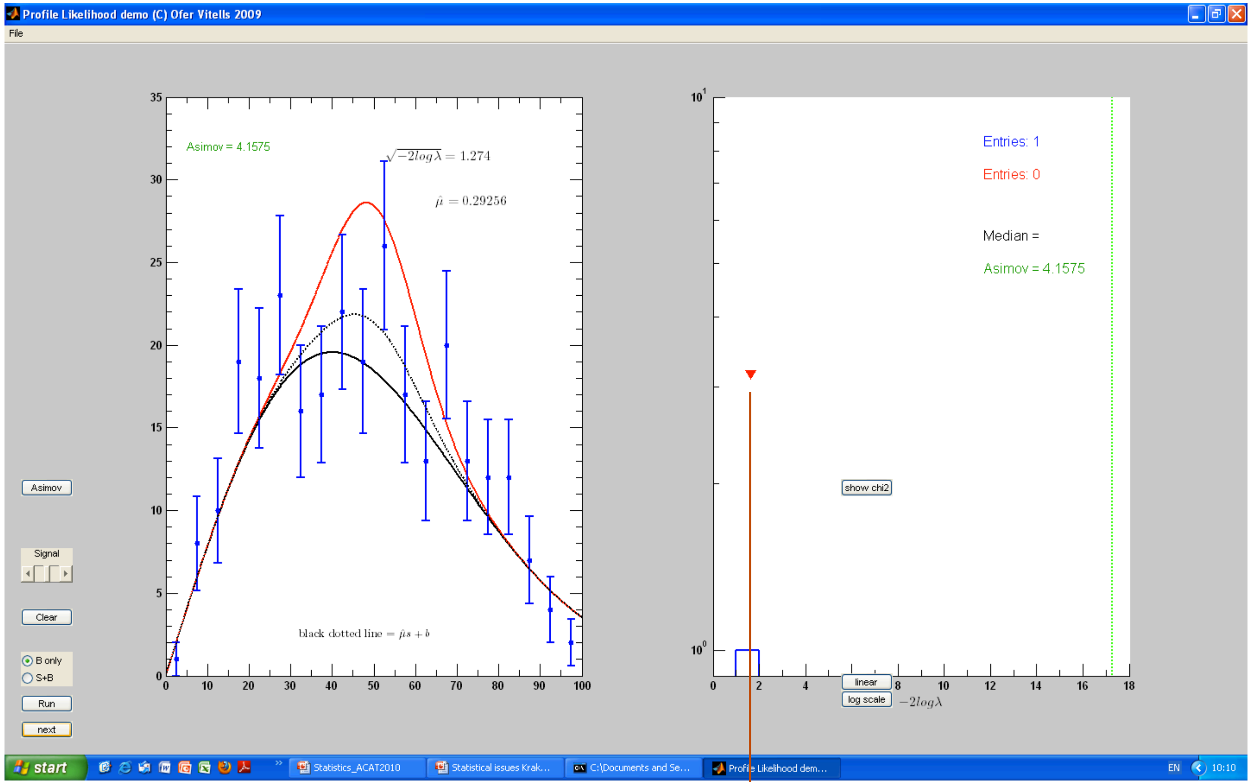
$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$

$$\hat{\mu} > 0$$

$\hat{\mu} \sim 0,\ q$ small

$\hat{\mu} \sim 1,\ q$ large

The best signal $\hat{\mu} = 0.3 \rightarrow 1.27\sigma$



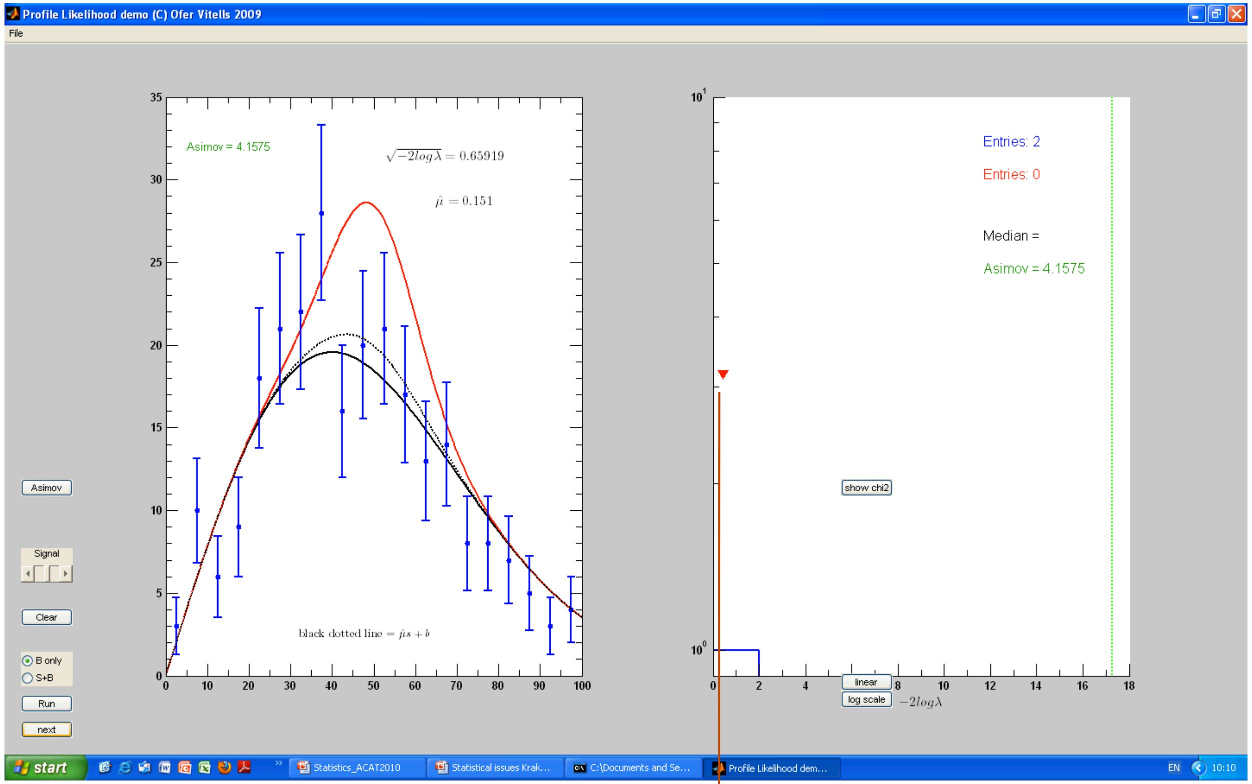
$$q = 1.6 \rightarrow Z = \sqrt{1.6} = 1.27$$

PL: test t under BG only ; $f(q_0 | H_0)$

$$\hat{\mu} = 0.15 \rightarrow 0.6\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s + b)}$$

$$\hat{\mu} > 0$$



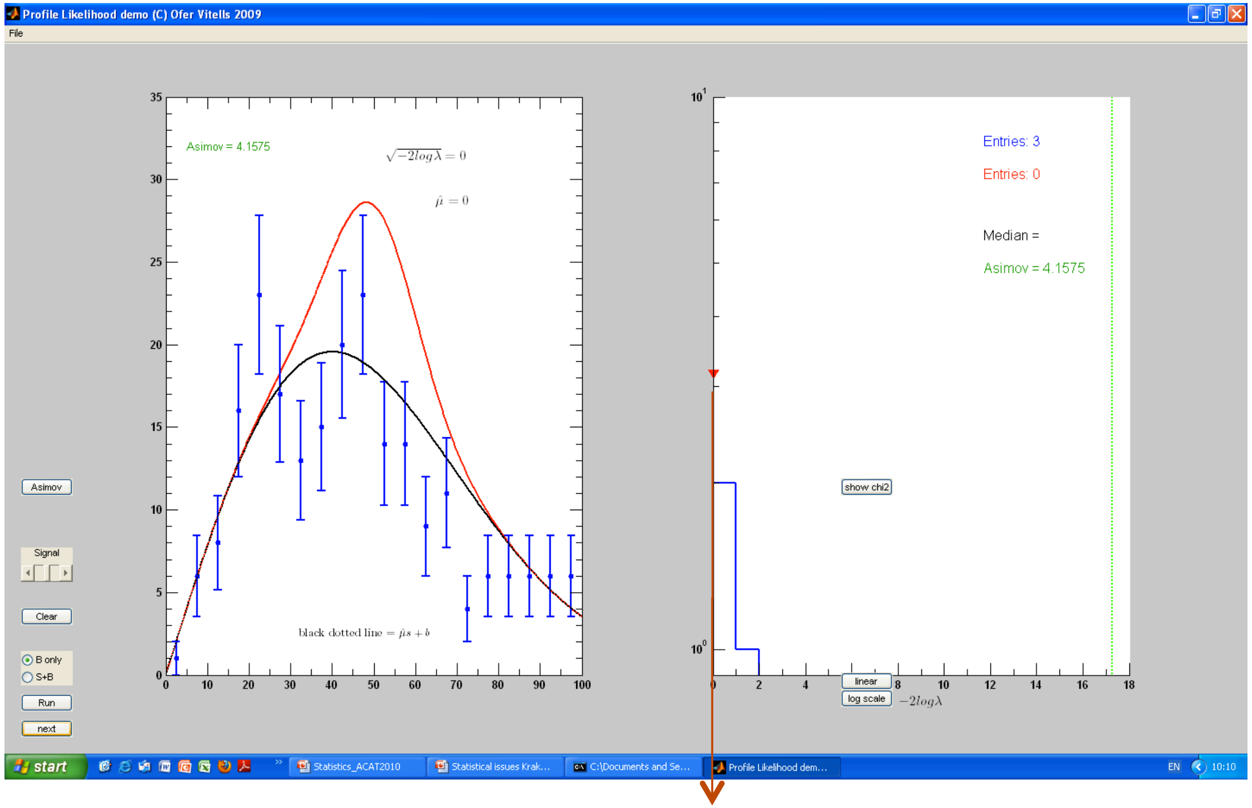
$$q = 0.43 \rightarrow Z = 0.66\sigma$$

PL: test t under BG only ; $f(q_0 | H_0)$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$

$$\hat{\mu} > 0$$

$$\hat{\mu} = 0$$



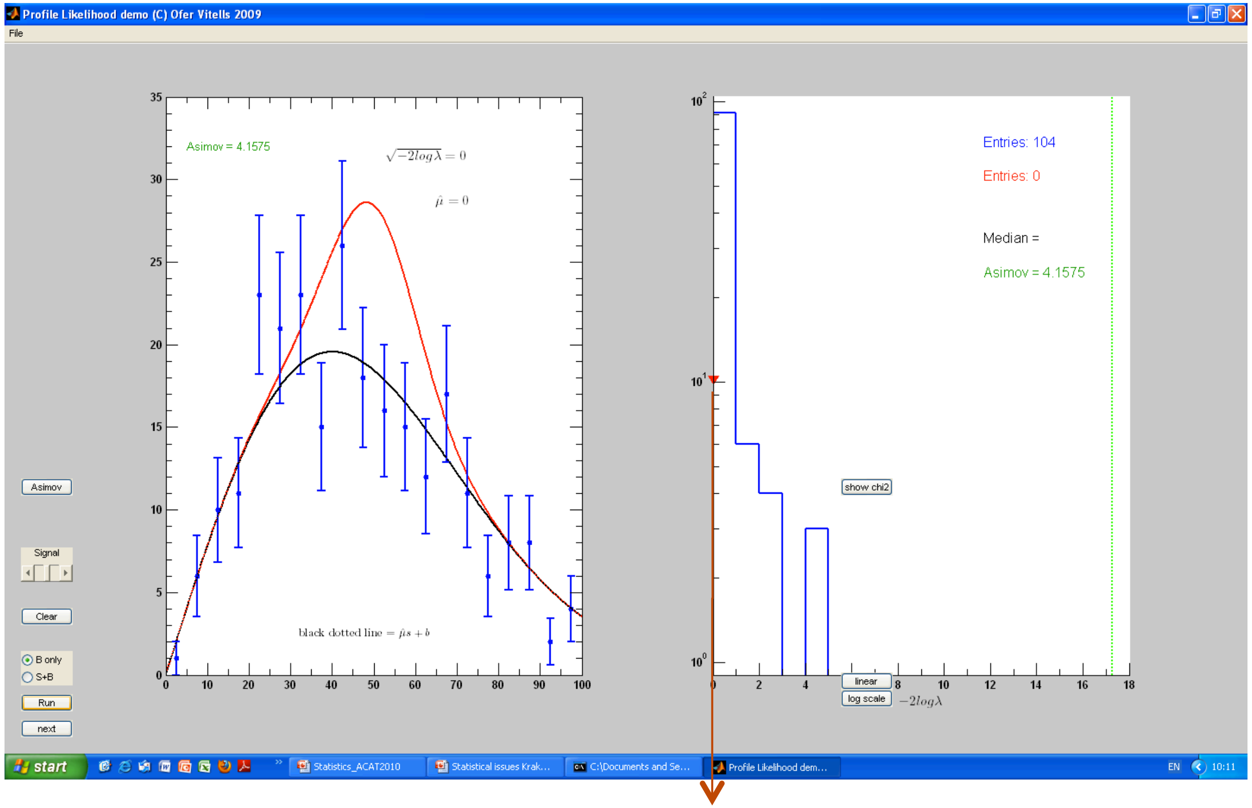
$$q = 0$$

PL: test t under BG only ; $f(q_0 | H_0)$

$\hat{\mu} = 0$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s + b)}$$

$$\hat{\mu} > 0$$



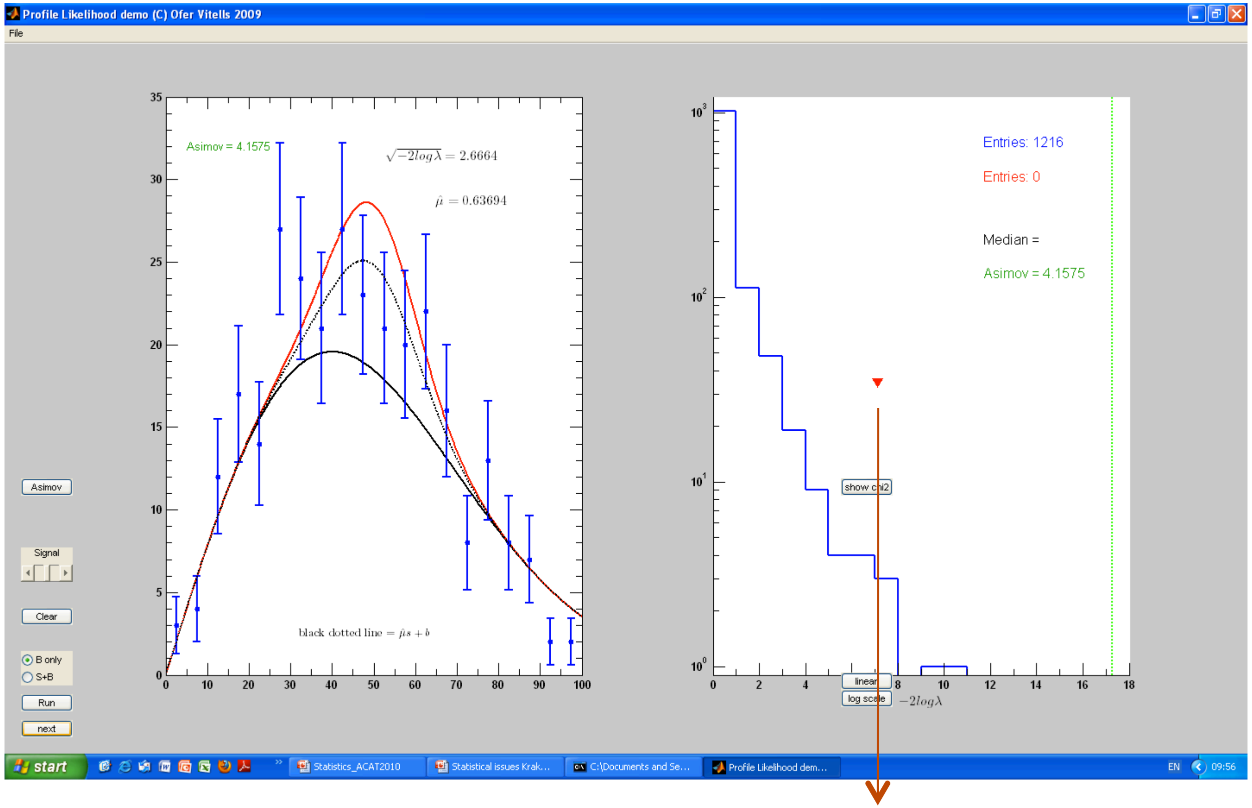
$q = 0$

PL: test t under BG only ; f(q$_0$|H$_0$)

$$\hat{\mu} = 0.6 \rightarrow 2.6\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$

$$\hat{\mu} > 0$$

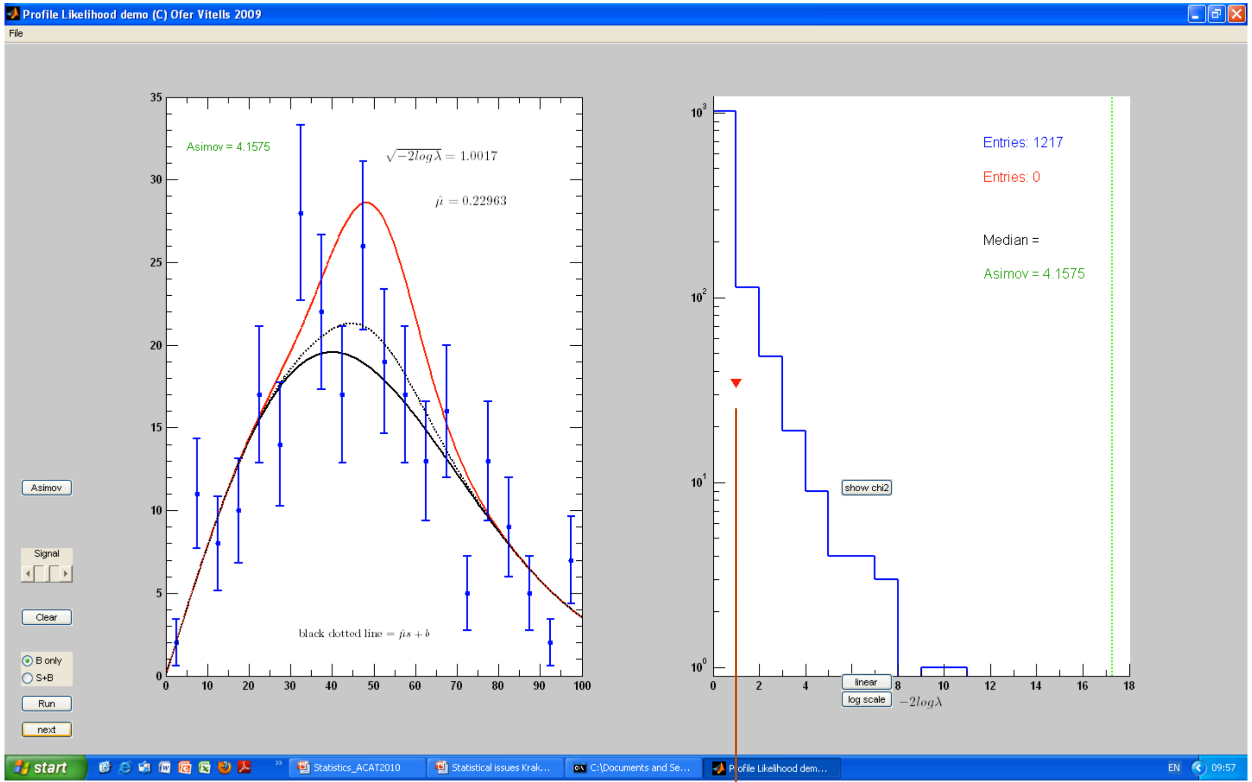


$$q = 6.76 \rightarrow Z = 2.6\sigma$$

PL: test t under BG only ; f(q_0|H_0)

$$\hat{\mu} = 0.22 \rightarrow 1.1\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$
$$\hat{\mu} > 0$$



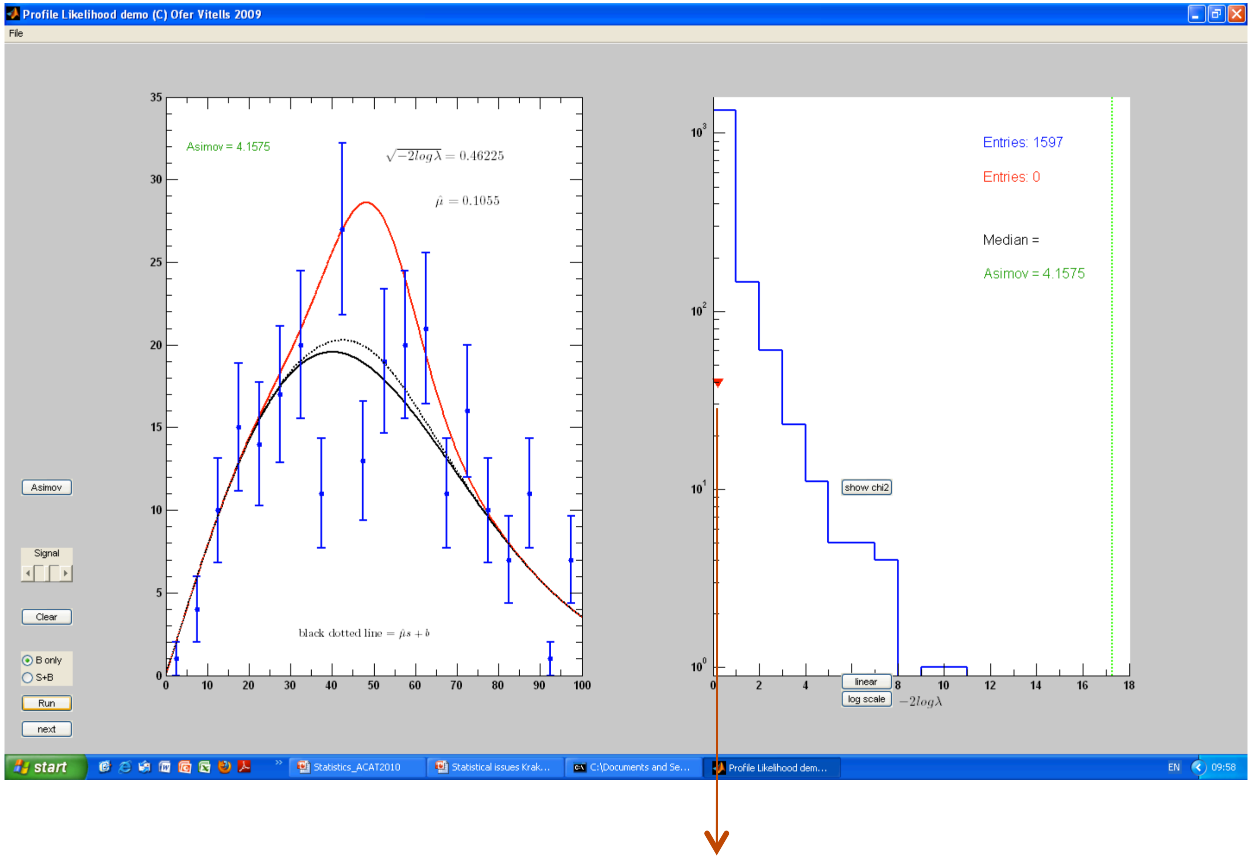
$$q = 1.2 \rightarrow Z = 1.1\sigma$$

PL: test t under BG only ; $f(t_0 | H_0)$

$$\hat{\mu} = 0.11 \rightarrow 0.4\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s + b)}$$

$$\hat{\mu} > 0$$



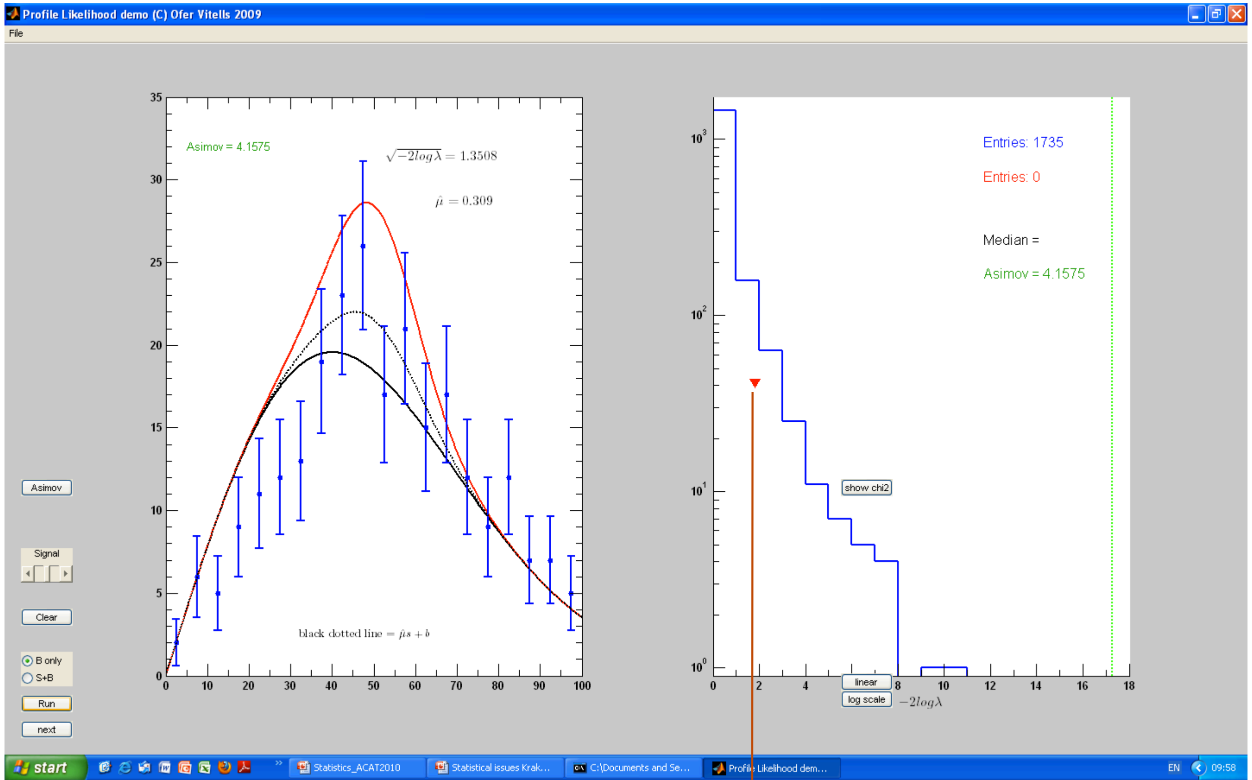
$$q = 0.16 \rightarrow Z = 0.4\sigma$$

PL: test t under BG only ; $f(q_0 | H_0)$

$$\hat{\mu} = 0.31 \rightarrow 1.35\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s + b)}$$

$$\hat{\mu} > 0$$



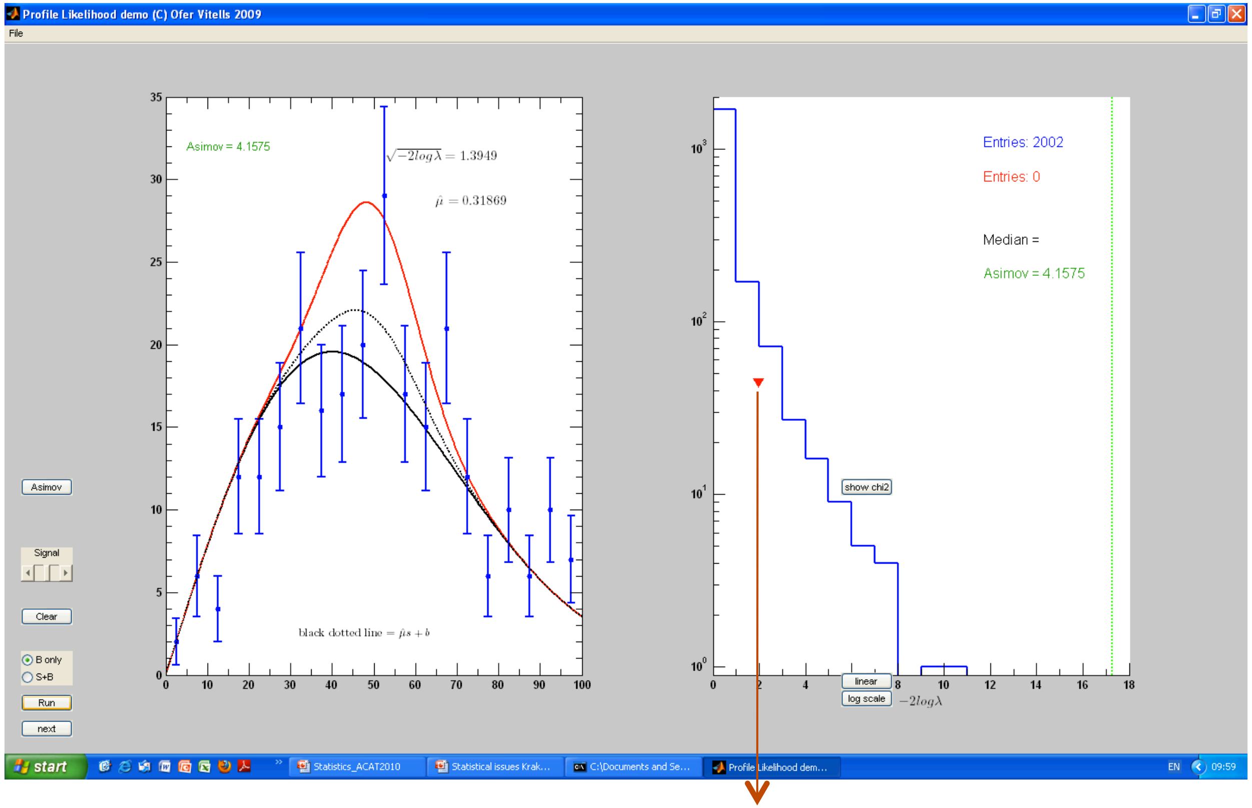
$$q = 1.8 \rightarrow Z = 1.35\sigma$$

PL: test t under BG only ; $f(q_0 | H_0)$

$$\hat{\mu} = 0.32 \rightarrow 1.39\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s + b)}$$

$$\hat{\mu} > 0$$



$$q = 1.9 \rightarrow Z = 1.39\sigma$$

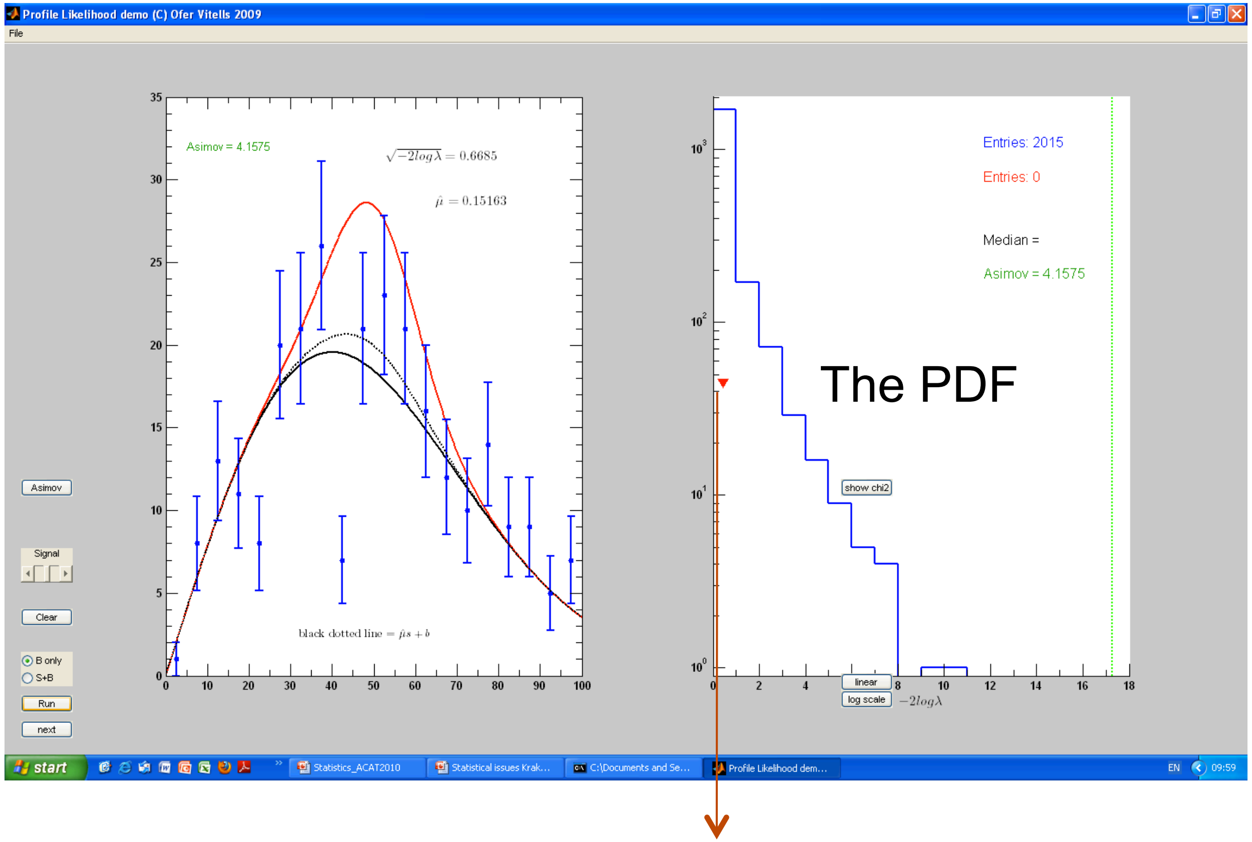
PL: test t under BG only ; $f(t_0 | H_0)$

$$\hat{\mu} = 0.15 \rightarrow 0.66\sigma$$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s + b)}$$

$$\hat{\mu} > 0$$

More than 10^6 toy MC experiments are needed to get the PDF to the level of 5σ



$$q = 0.43 \rightarrow Z = 0.66\sigma$$

Wilks Theorem

S.S. Wilks, *The large-sample distribution of the likelihood ratio for testing composite hypotheses*, Ann. Math. Statist. **9** (1938) 60-2.

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}\cdot s+b)}$$

- Under a set of regularity conditions and for a sufficiently large data sample, *Wilks' theorem says that* the pdf of the statistic q under the null hypothesis approaches a chi-square PDF for one degree of freedom $f(q_0 \mid H_0) = \chi_1^2$
- Same token $q_1 = -2\ln\frac{L(s+b)}{L(\hat{\mu}\cdot s+b)}$ $f(q_1 \mid H_1) \sim \chi_1^2$

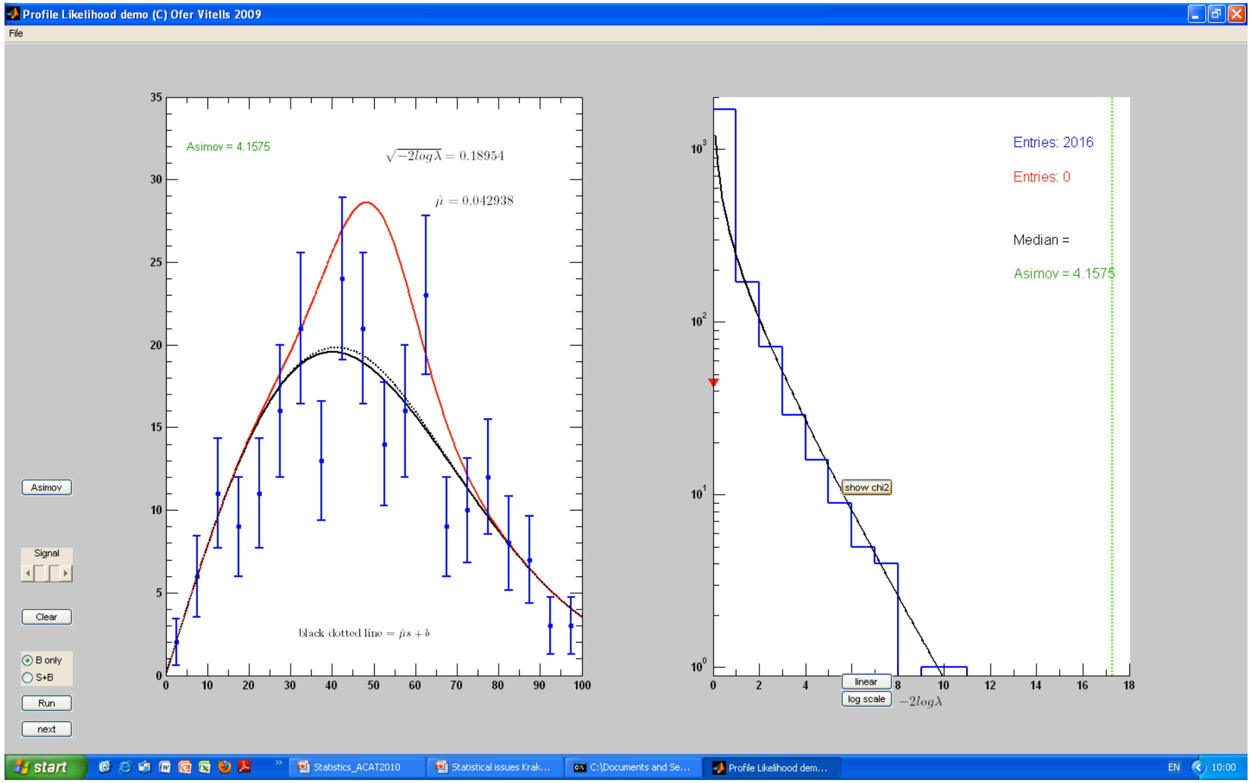
Wilks Theorem

• For the test statistic $q_\mu = -2\ln\frac{L(\mu)}{L(\hat{\mu})}$; $q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$ $\hat{\mu} > 0$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)}$$

$$f(q_0 \mid H_0) = \chi_1^2$$

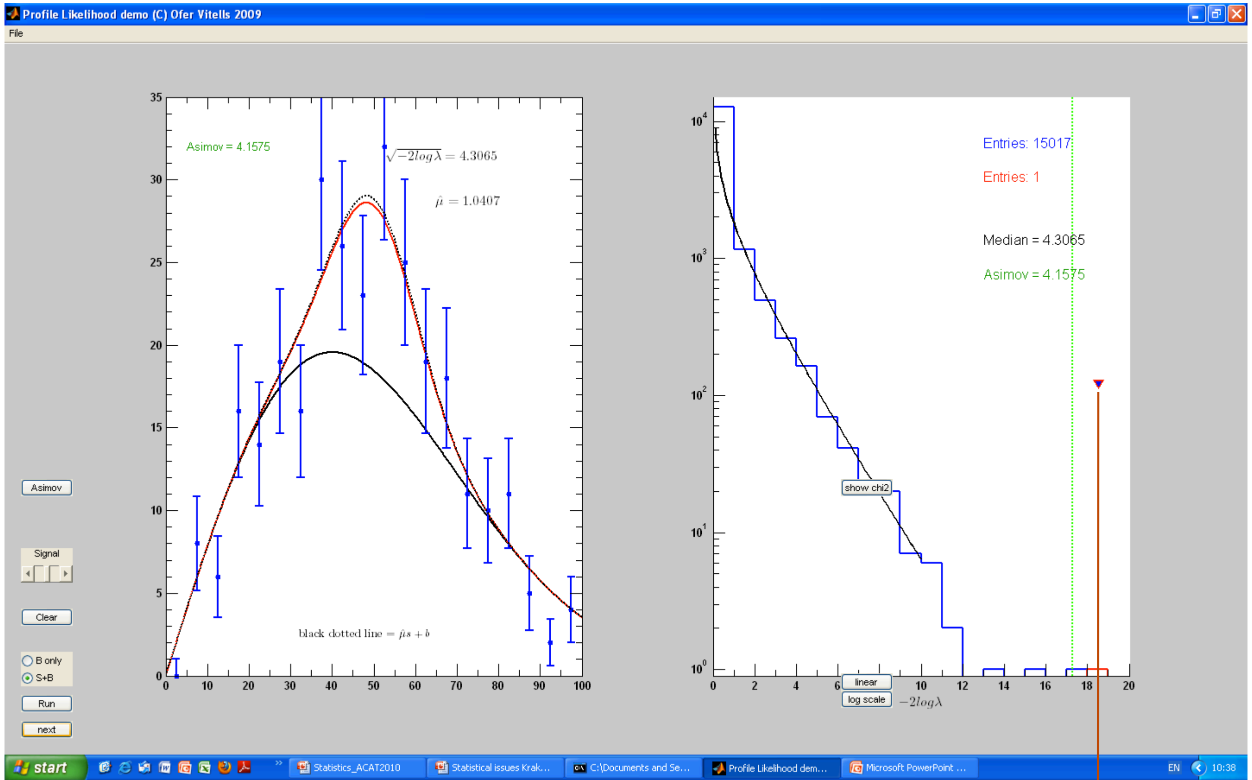
$$f(q_\mu \mid H_\mu) = \chi_1^2$$



The PDF of q under s+b experiments (H_1)

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)} = -2\ln\frac{L(b\,|\,H_1)}{L(\hat{\mu}s+b\,|\,H_1)} \quad \hat{\mu} > 0$$

$$\hat{\mu} = 1.04 \rightarrow 4.3\sigma$$



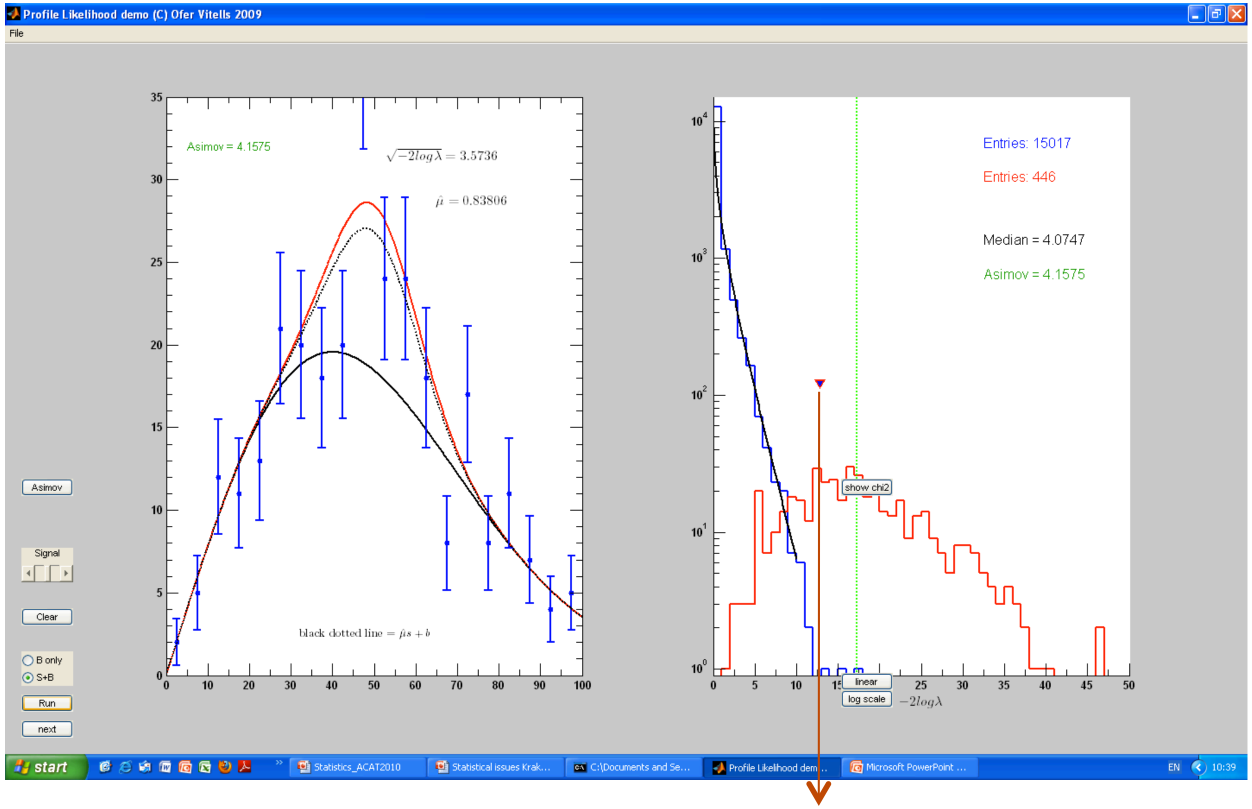
$$q = 18.5 \rightarrow Z = 4.3\sigma$$

PL: test q_1 under s+b; $f(q_0|H_1)$

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)} = -2\ln\frac{L(b|H_1)}{L(\hat{\mu}s+b|H_1)} \quad \hat{\mu} > 0$$

$$\hat{\mu} = 0.83 \rightarrow 3.6\sigma$$

- Can we quantify the sensitivity of an experiment for discovery?
- Can we formulate mathematically the PDF of the alternate hypothesis?

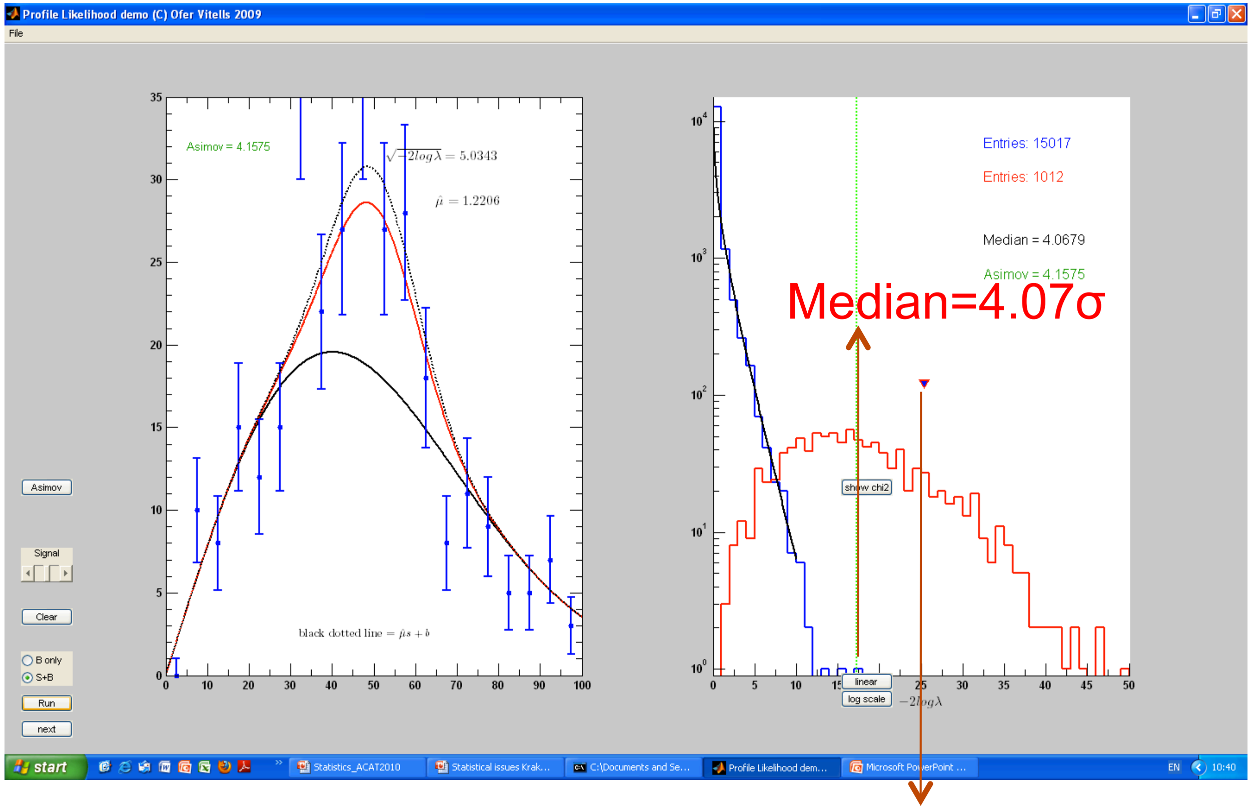


$$q = 12.9 \rightarrow Z = 3.6\sigma$$

Expected Discovery Sensitivity

$$q_0 = -2\ln\frac{L(b)}{L(\hat{\mu}s+b)} = -2\ln\frac{L(b\,|\,H_1)}{L(\hat{\mu}s+b\,|\,H_1)} \quad \hat{\mu} > 0$$

$$\hat{\mu} = 1.22 \rightarrow 5.0\sigma$$

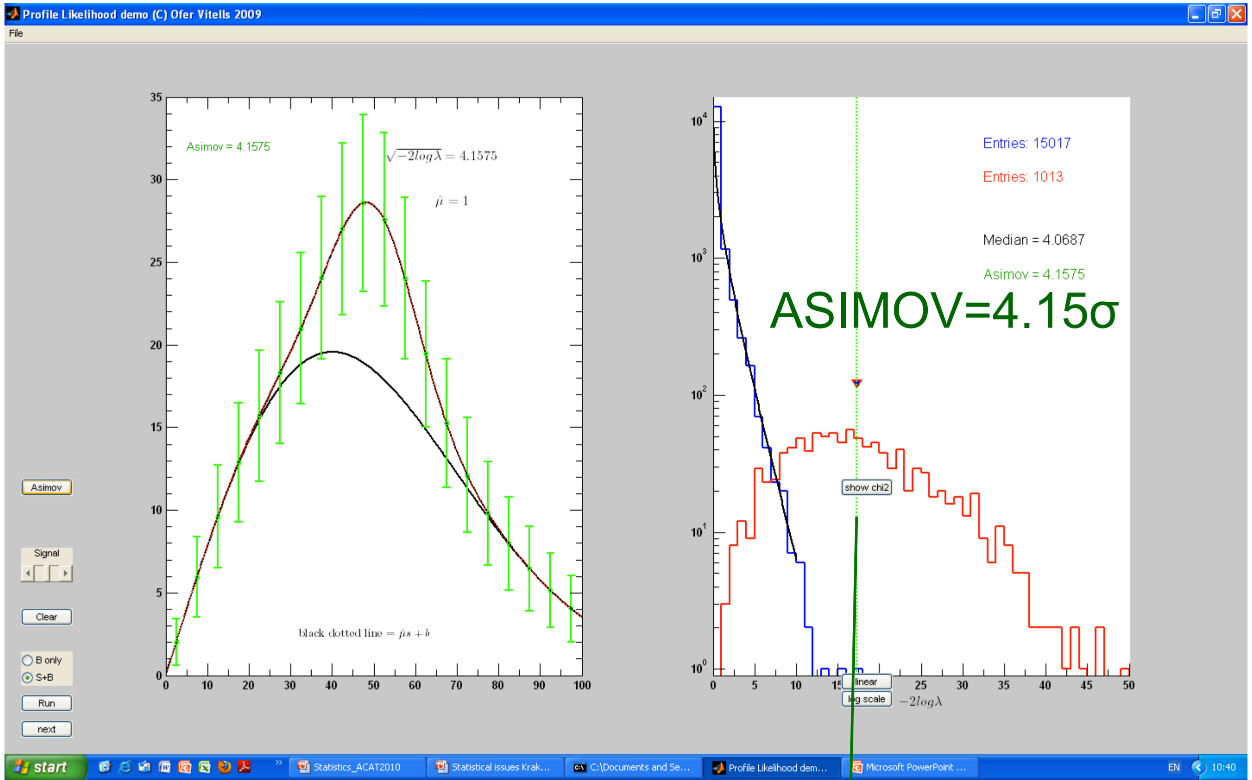


$$q = 25 \rightarrow Z = 5.0\sigma$$

The Median Sensitivity (via ASIMOV)

$$\hat{\mu} = 1.00 \rightarrow 4.15\sigma$$

- To estimate the median sensitivity of an experiment (**before looking at the data**), one can either perform lots of s+b experiments and estimate the median $q_{o,med}$ or evaluate q_0 with respect to a representative data set, the ASIMOV data set with μ=1, i.e. x=s+b



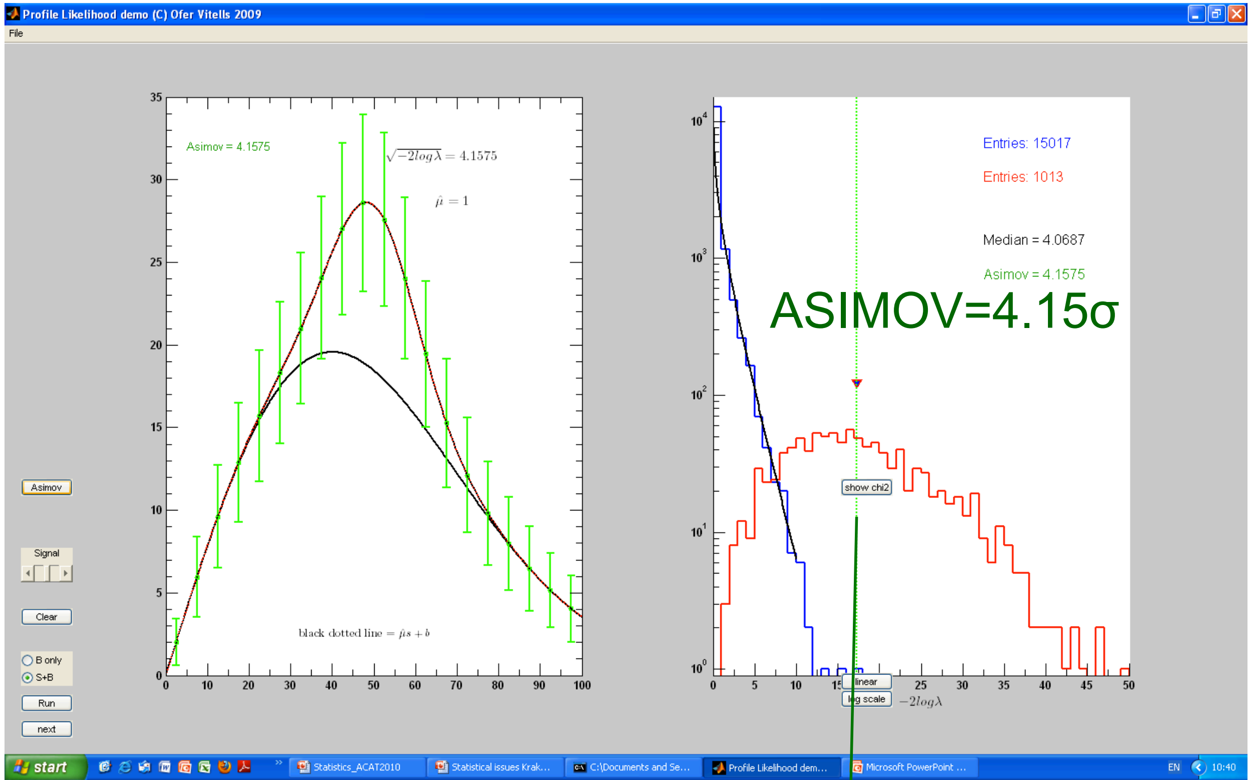
$$q_A = 17.22 \rightarrow Z_A = 4.15$$

$$q_{o,med} \simeq q_0(\hat{\mu} = \mu_A = 1) = -2\ln\frac{L(b \mid x = x_A = s+b)}{L(\hat{\mu}s + b \mid x = x_A = s+b)} = -2\ln\frac{L(b)}{L(1 \cdot s + b)}$$

The Median Sensitivity (via ASIMOV)

$$\hat{\mu} = 1.00 \rightarrow 4.15\sigma$$

- The name of the Asimov data set is inspired by the short story Franchise, by Isaac Asimov. In it, elections are held by selecting the single most representative voter to replace the entire electorate.



$$q_A = 17.22 \rightarrow Z_A = 4.15$$

$$q_{o,med} \simeq q_0(\hat{\mu} = \mu_A = 1) = -2\ln\frac{L(b \mid x = x_A = s+b)}{L(\hat{\mu}s + b \mid x = x_A = s+b)} = -2\ln\frac{L(b)}{L(1\cdot s+b)}$$

Nuisance Parameters

- Normally, the background, $b(\theta)$, has an uncertainty which has to be taken into account. In this case θ is called a nuisance parameter (which we associate with background systematics)
- The signal strength μ is a parameter of interest
- How can we take into account the nuisance parameters?

$$q_\mu = -2\log\frac{\max_\theta L(\mu,\theta)}{\max_{\mu,\theta} L(\mu,\theta)} = -2\log\frac{L(\mu,\hat{\hat{\theta}}_\mu)}{L(\hat{\mu},\hat{\theta})}$$

$$\hat{\hat{\theta}}_\mu : MLE\ of\ \ L(\mu s + b(\hat{\hat{\theta}})), \qquad \hat{\mu},\hat{\theta} : MLE\ of\ \ L(\hat{\mu} s + b(\hat{\theta}))$$

- The PDF $f(q_\mu \mid H_\mu)$ is asymptotically not dependent on the nuisance parameters, that's a big advantage of the PL

Asymptotic formulae for likelihood-based tests of new physics

Cowan, Cranmer, Gross, Vitells, arXiv:1007.1727,
EPJC 71 (2011) 1-19

Significance test using profile likelihood ratio

Systematics included via nuisance parameters

Distributions in large sample limit, no MC used.

http://arxiv.org/abs/1007.1727v2

Asymptotic distributions of q_0 (=t_0)

- Wilks

$$f(q_0|0) \sim \frac{1}{2}\chi^2$$

The factor half due to the one sided $\hat{\mu} > 0$
[H. Chernoff, Ann. Math. Stat. 25, 573578 (1954)]

$$f(q_0|0) = \frac{1}{2}\delta(q_0) + \frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{q_0}}e^{-q_0/2}$$

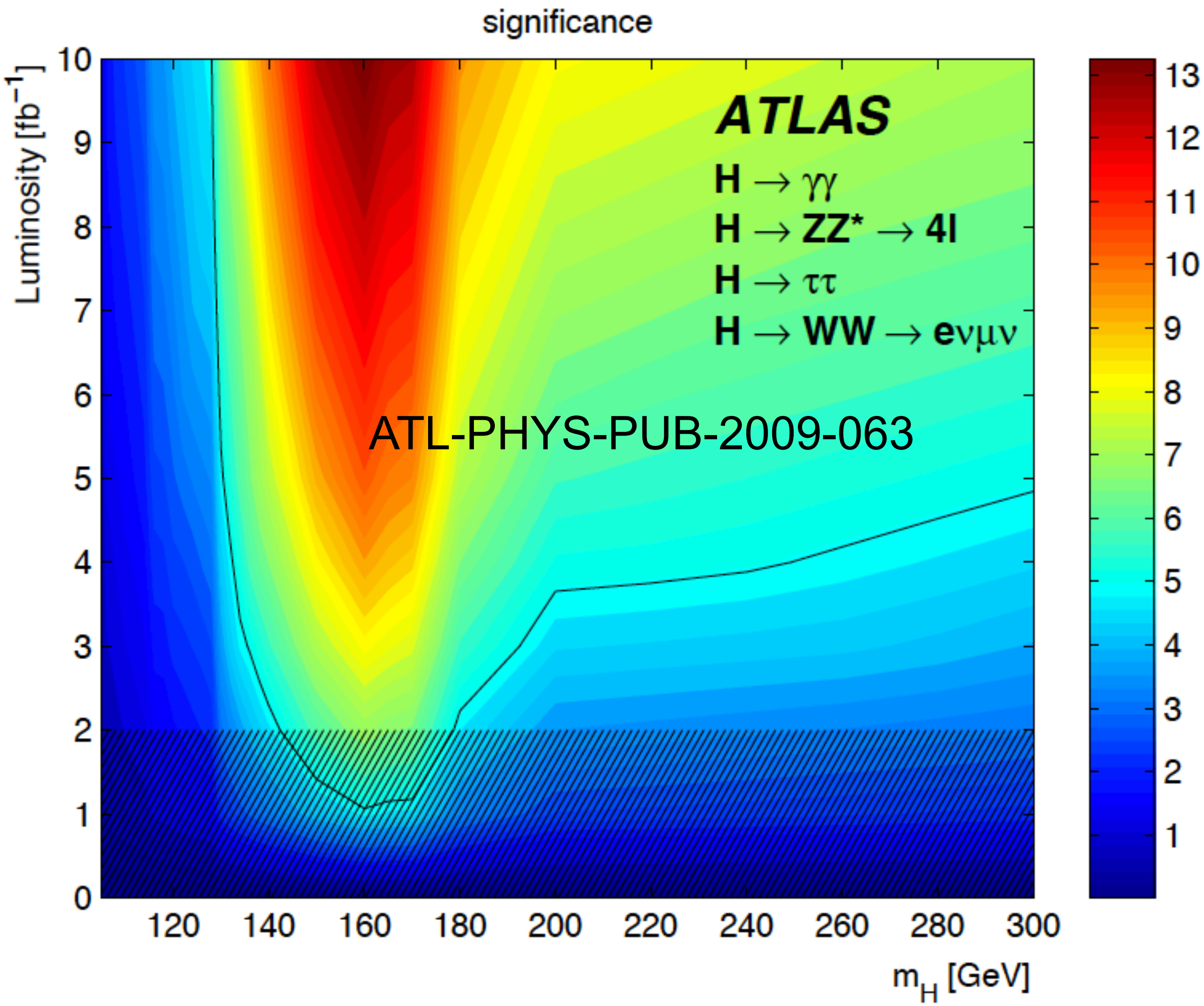
- Walds $f(q_0|\mu') \sim \chi^2(\Lambda = \frac{\mu'^2}{\sigma^2})$

$$f(q_0|\mu') = \left(1 - \Phi\left(\frac{\mu'}{\sigma}\right)\right)\delta(q_0) + \frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{q_0}}\exp\left[-\frac{1}{2}\left(\sqrt{q_0} - \frac{\mu'}{\sigma}\right)^2\right]$$

$$\sqrt{(q_0|data)} = Z$$

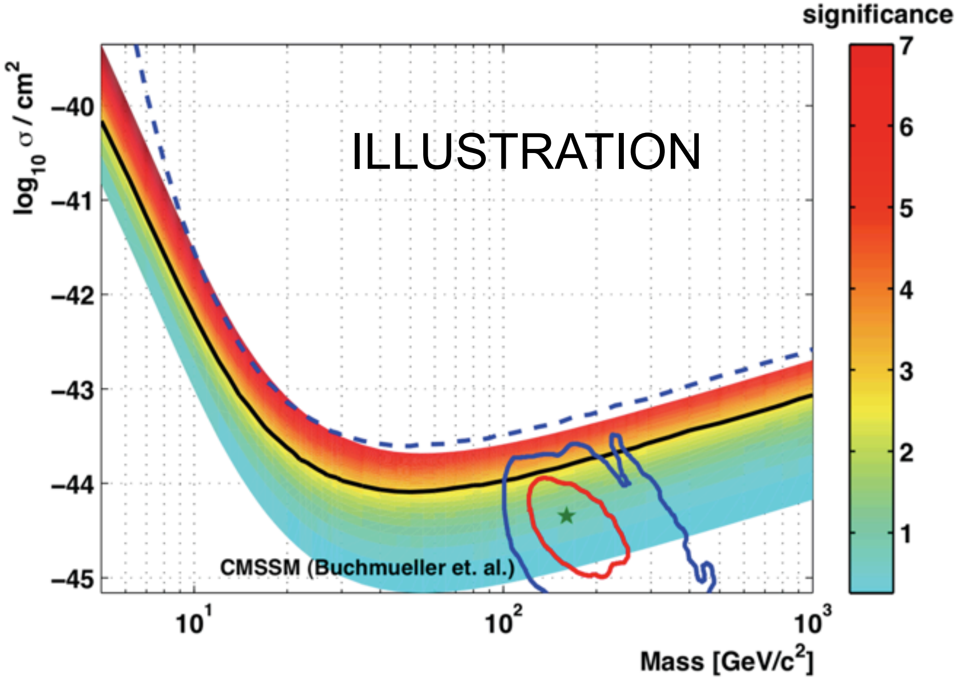
$$\sqrt{(q_0|Asimov = \mu' s + b)} = med(Z)$$

Application to HEP: Expected Higgs sensitivity at 14 TeV



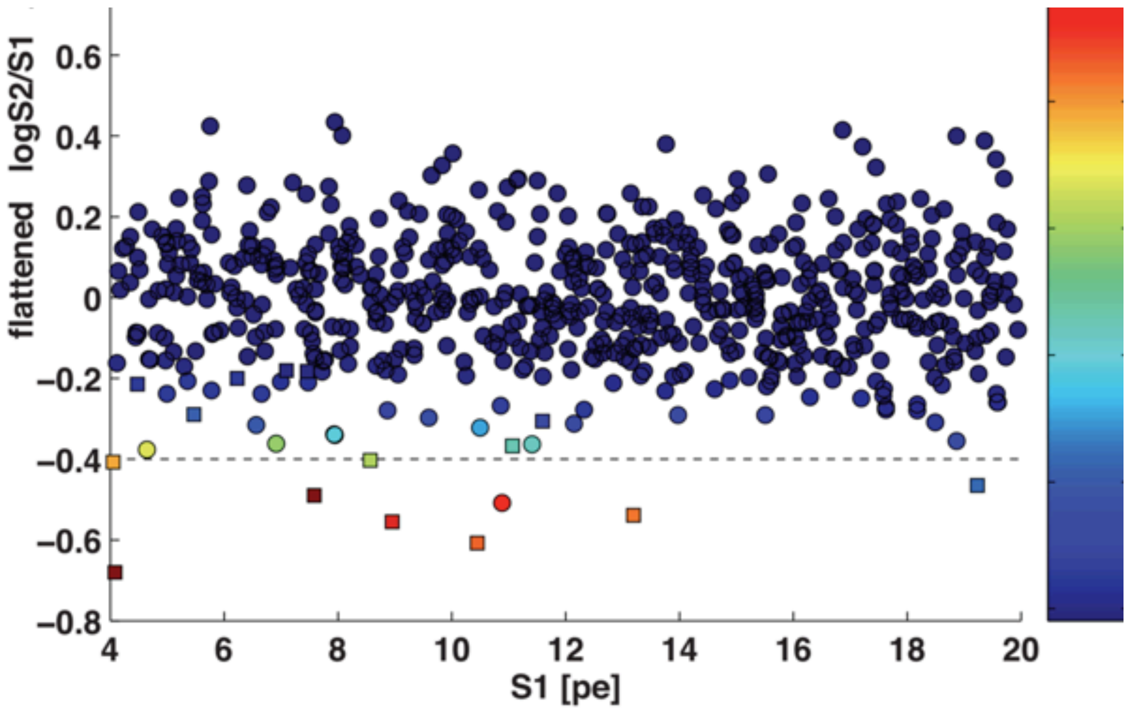
Application to DM:
WIMP Discovery Potential with XENON100

- By testing the background only hypothesis and trying to reject it, we can discover WIMPs with the PL method
- This is an illustration with 100 days of XENON 100
- SUSY contours take latest LHC results arXiv: 1102.4585



Discover WIMPs with PL

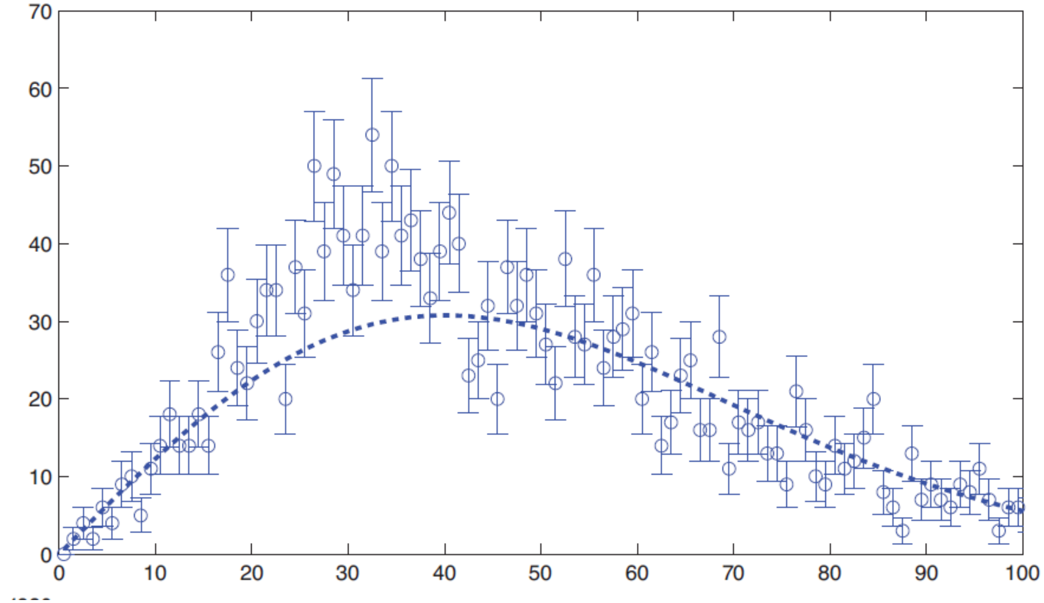
- An illustration showing the power of PL in testing for Discovery
- WIMPS (squares) are injected into a BG sample.
- By removing events and check the $\Delta(p-value)$ we can score events
- The colors represent significance of the events



The Look Elsewhere Effect

Look Elsewhere Effect

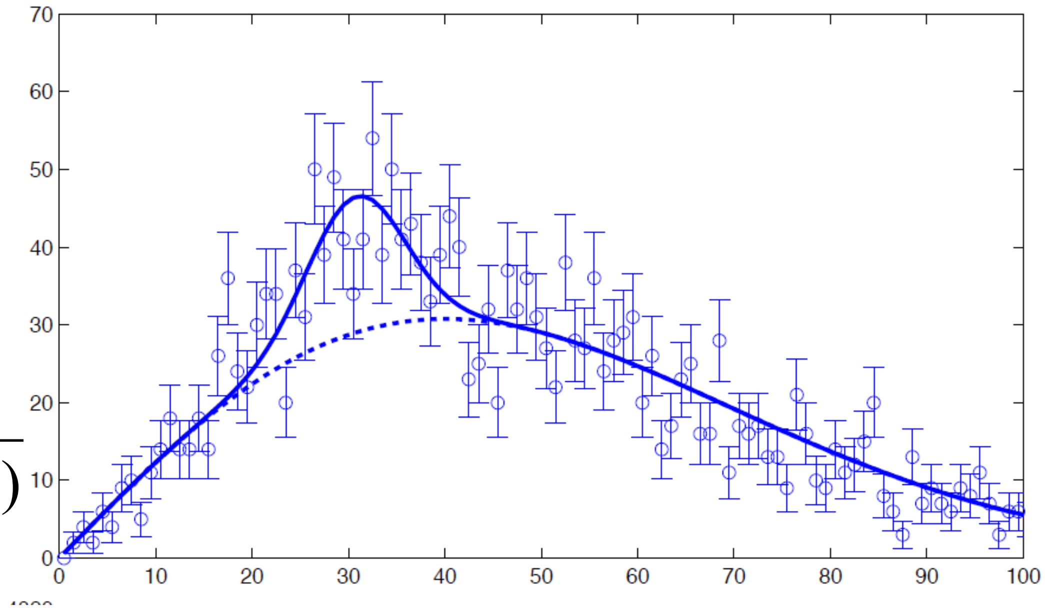
- Is there a signal here?



Look Elsewhere Effect

- Obviously @ m=30
- What is its significance?
- What is your test statistic?

$$q_{fix,obs} = -2\ln\frac{L(b)}{L(\hat{\mu}s(m=30)+b)}$$



$$\hat{\mu} > 0$$

Look Elsewhere Effect

- Test statistic

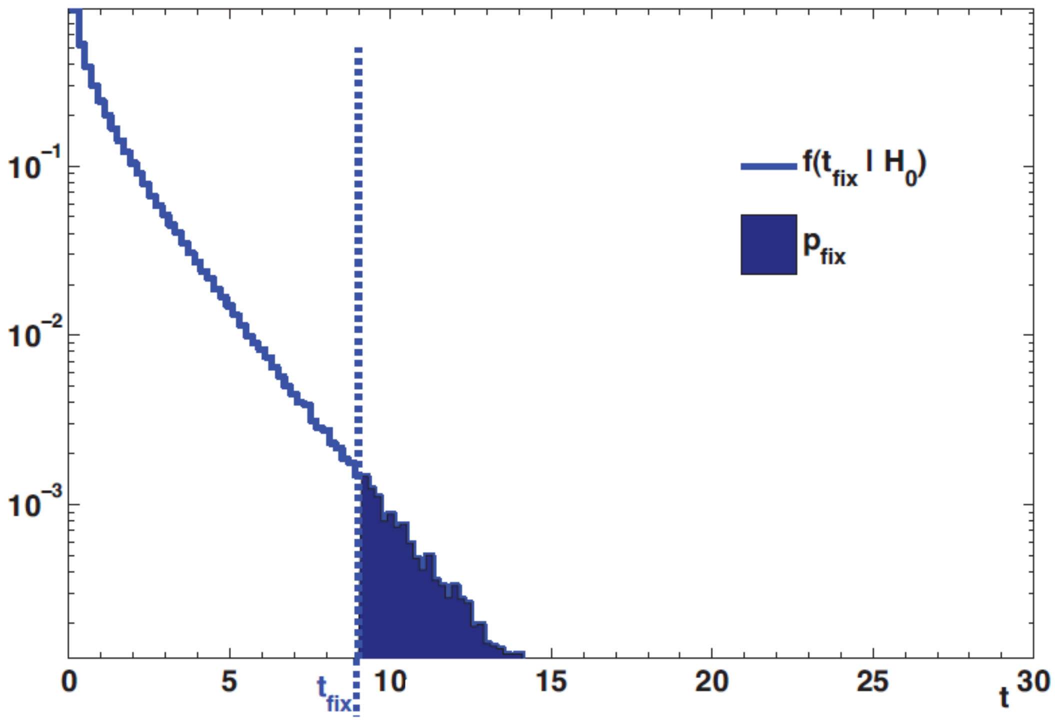
$$q_{fix,obs} = -2\ln\frac{L(b)}{L(\hat{\mu}s(m=30)+b)}$$

$$\hat{\mu} > 0$$

- What is the p-value?
- generate the PDF

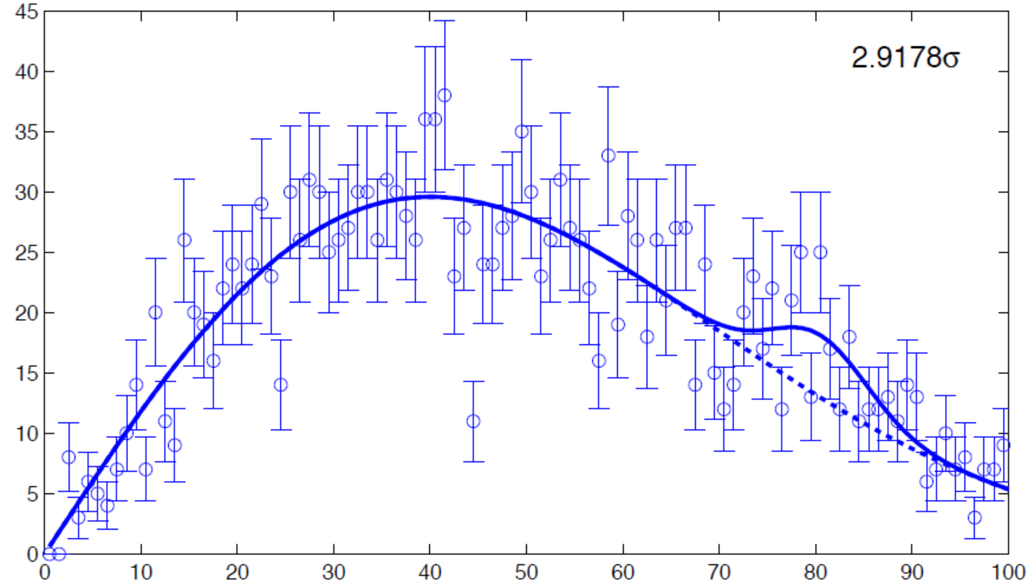
$$f(q_{fix} \mid H_0)$$

and find the **p-value**



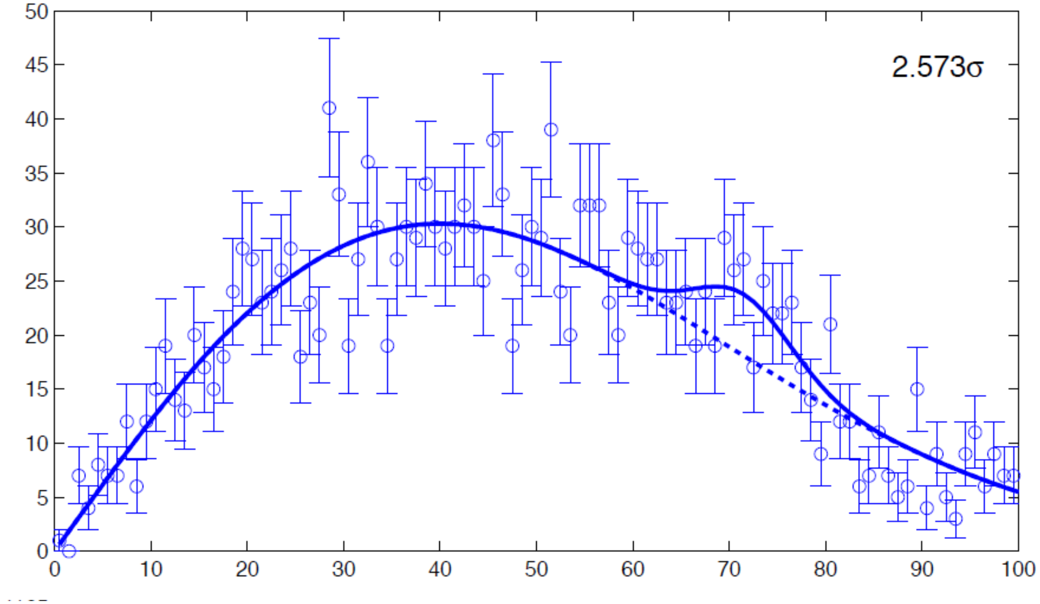
Look Elsewhere Effect

- Would you ignore this signal, had you seen it?



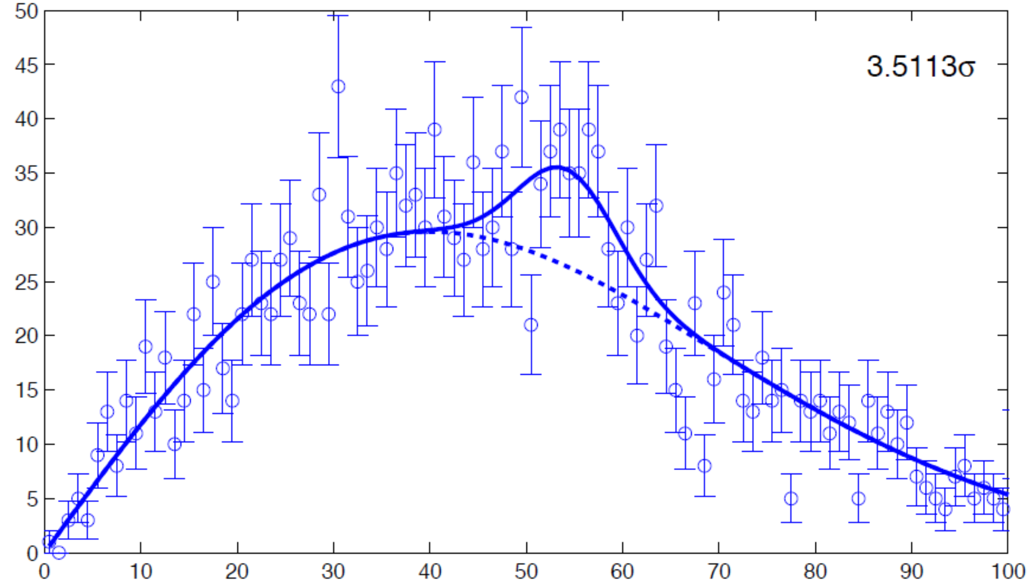
Look Elsewhere Effect

- Or this?



Look Elsewhere Effect

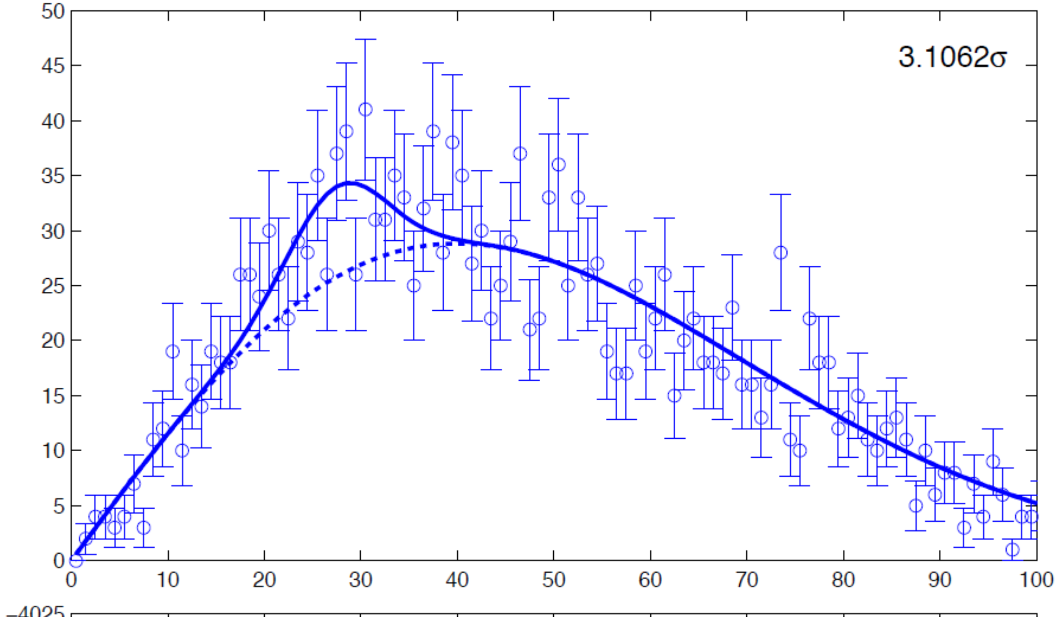
- Or this?



Look Elsewhere Effect

- Or this?

- Obviously NOT!

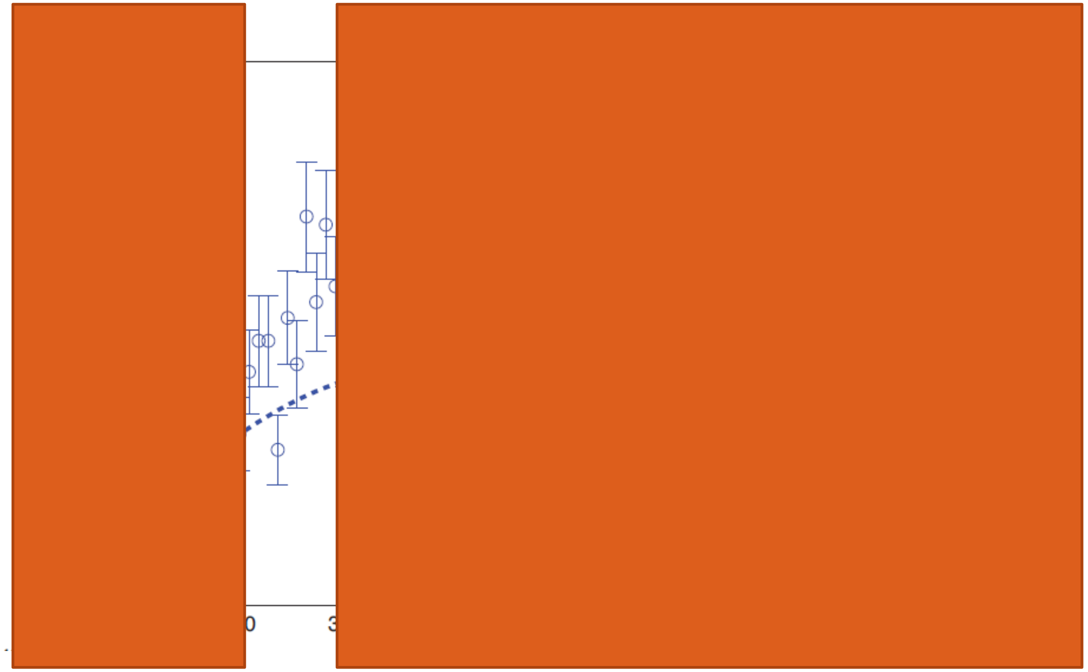


Look Elsewhere Effect

- Having no idea where the signal might be there are two options
- **OPTION I:**

scan the mass range in pre-defined steps and test any disturbing fluctuations
(Do let the facts confuse you :-)

- Perform a fixed mass analysis at each point



$$q_{fix,obs}(\hat{\mu}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(m)+b)}$$

$$\hat{\mu} > 0$$

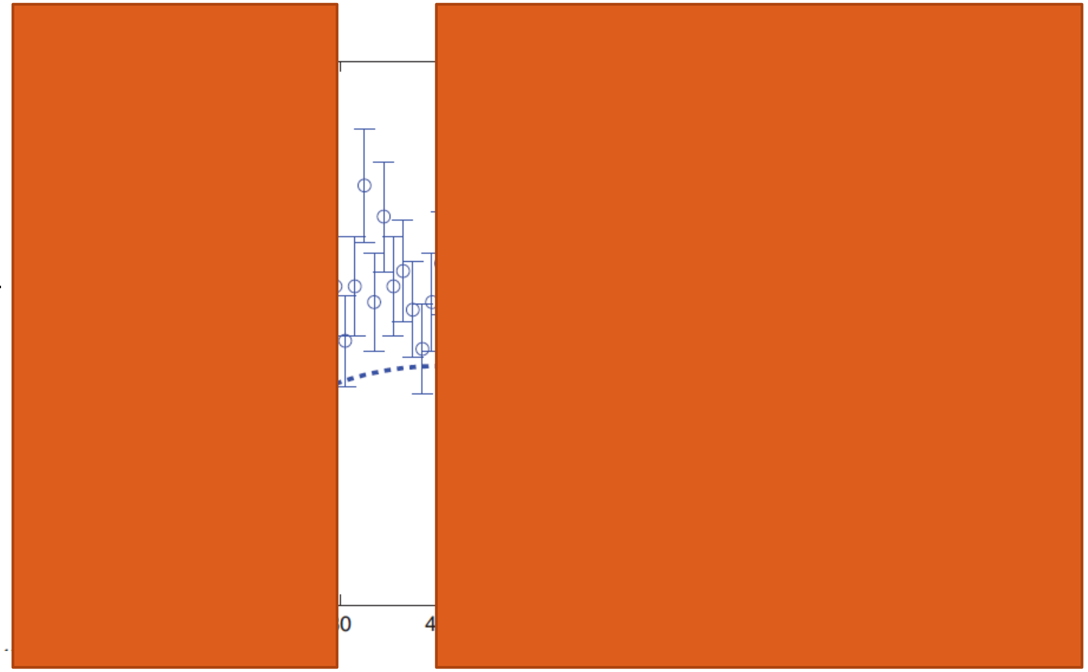
Look Elsewhere Effect

- Having no idea where the signal might be there are two options
- **OPTION I:**

scan the mass range in pre-defined steps and test any disturbing fluctuations

(Do let the facts confuse you :-)

- Perform a fixed mass analysis at each point



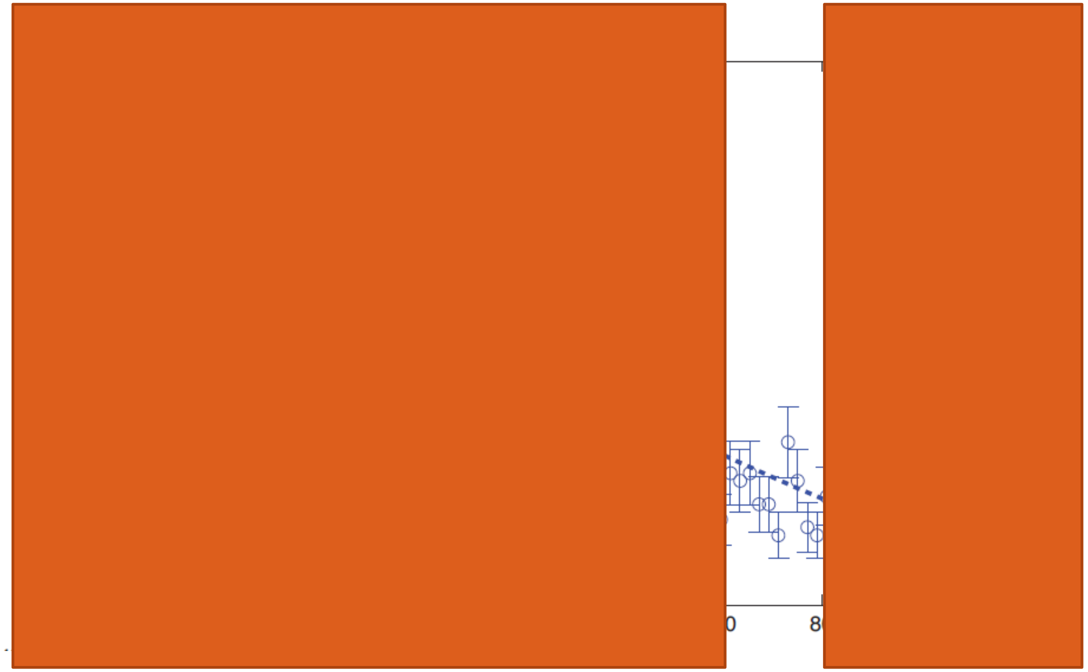
$$q_{fix,obs}(\hat{\mu}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(m)+b)}$$

$$\hat{\mu} > 0$$

Look Elsewhere Effect

- Of course the real signal might split in two windows
- The scan resolution must be less than the signal mass resolution

Assuming the signal can be only at one place, pick the one with the smallest p-value (maximum significance)



- This is equivalent to leaving the mass floating

$$\hat{q}_0 \equiv q_0(\hat{m}) = -2\log\frac{\mathcal{L}(\mu=0)}{\mathcal{L}(\hat{\mu},\hat{m})} = \max_m[q_{0,fix}(m)]$$

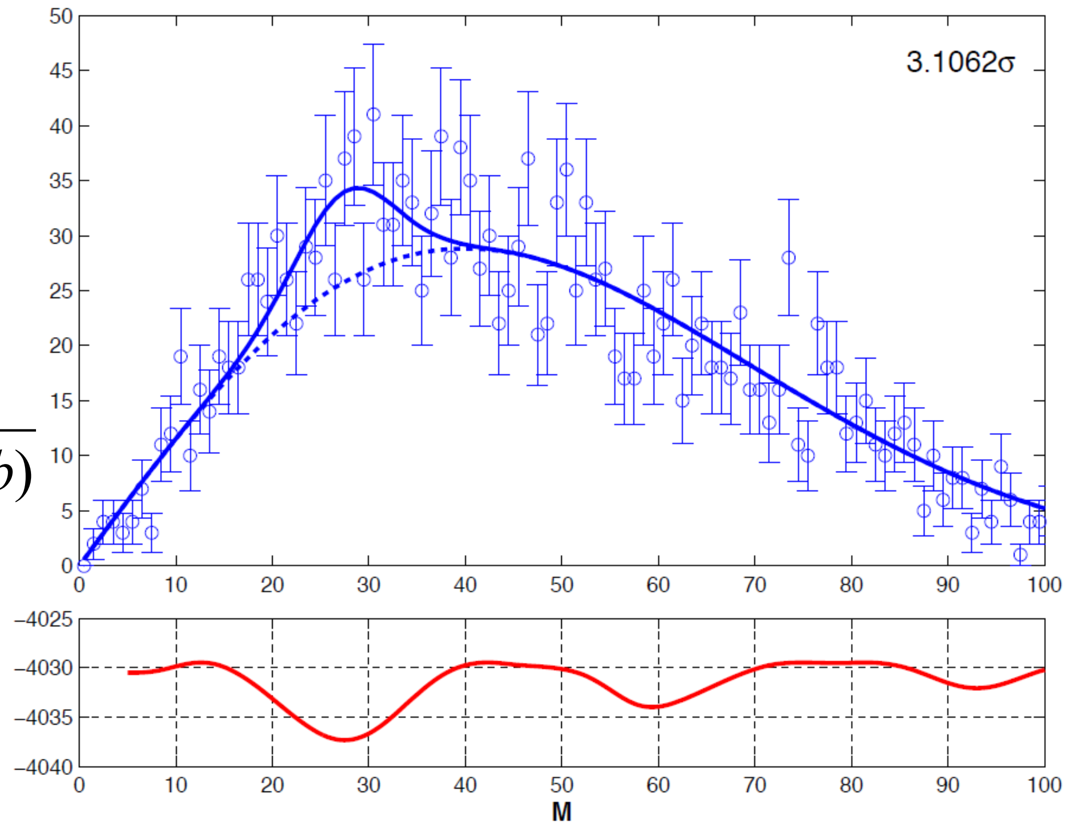
$$\hat{\mu} > 0$$

Look Elsewhere Effect

- Having no idea where the signal might be you would allow the signal to be anywhere in the **search range** and use a modified test statistic

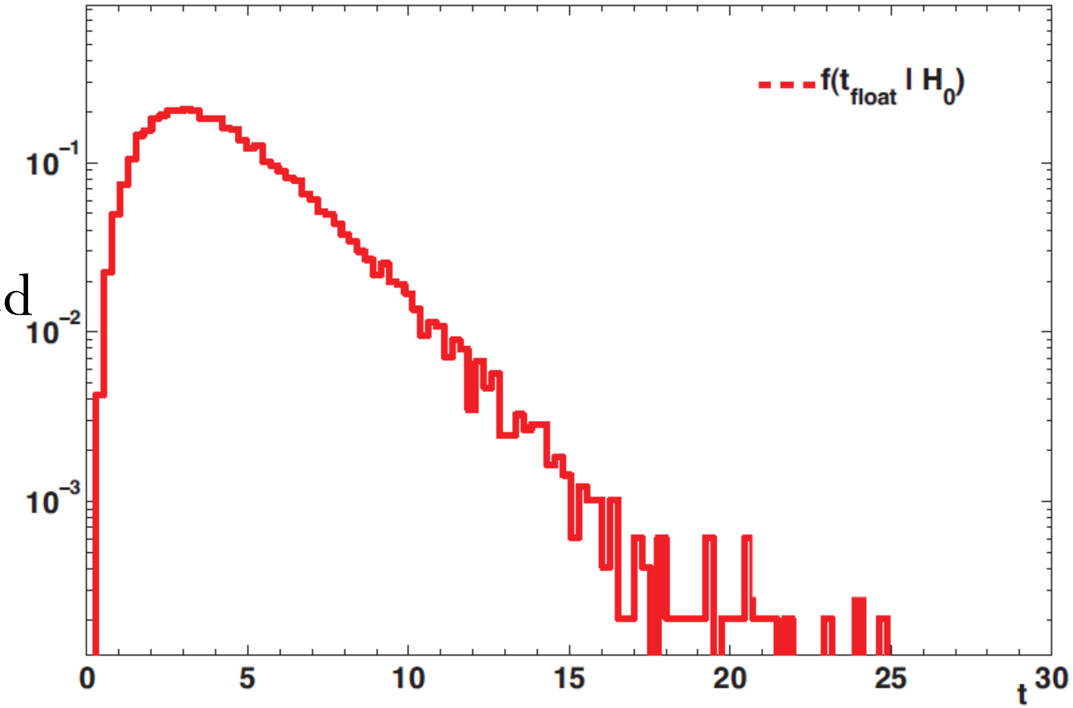
$$q_{float,obs}(\hat{\mu},\hat{m}) = -2\ln\frac{L(b)}{L(\hat{\mu}s(\hat{m})+b)}$$

- The p-value increases because more possibilities are opened



Look Elsewhere Effect

- The test statistic $q_{float,obs}(\mu=0) \equiv \hat{q}_0 = q_0(\hat{m}) = -2\ln\frac{L\left((\mu=0)\cdot \mathrm{s(m)+b}\right)}{L(\hat{\mu}s(\hat{m})+b)}$ has a special feature: It has a nuisance parameter which is not well defined under the null hypothesis (m) →
- The null hypothesis PDF $f(q_{float} \mid H_0)$ does not follow a chi-squared with 1 or 2 dof
- There are multiple minima depending on the size of the search range

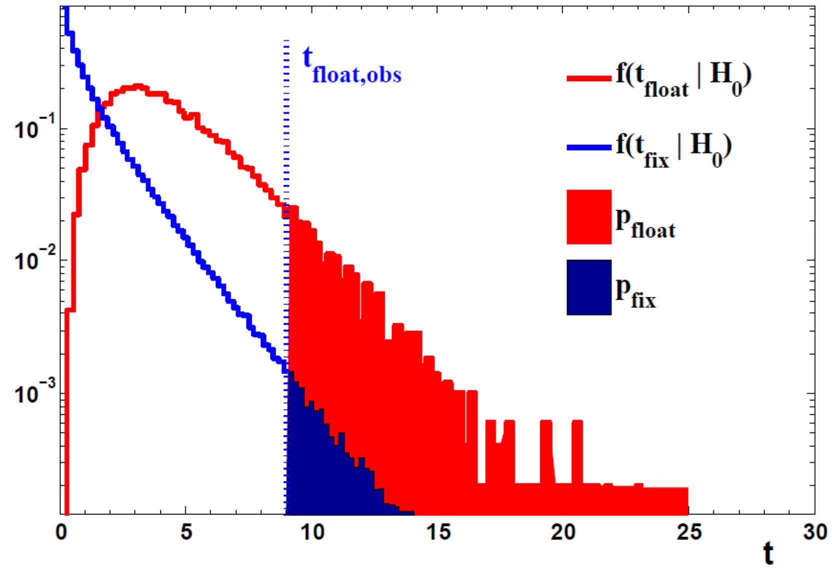


Look Elsewhere Effect

- We can now ask the question: Assume the Higgs is observed at some mass $\hat{m}$ what is the probability for the background to fluctuate locally at the observed level (or more)

$$@ m_H = \hat{m}$$

$$q_{fix,obs} = q_{float,obs} = -2\ln\frac{L(b)}{L(\hat{\mu}s(\hat{m}=m)+b)}$$



- We can calculate the following p-value

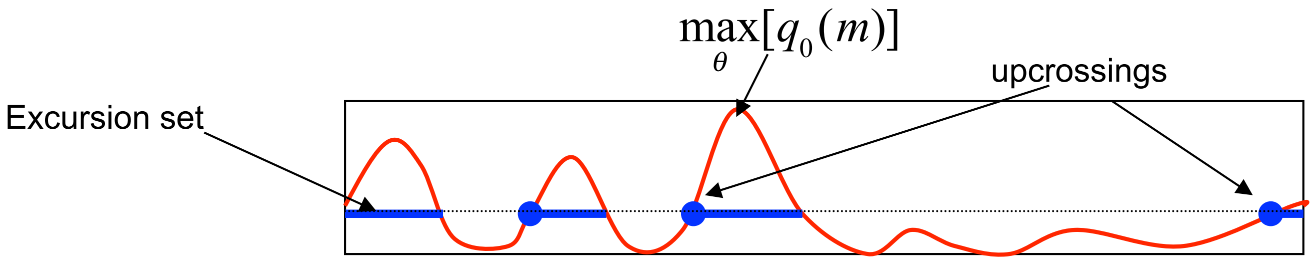
$$p_{fix} = \int_{t_{obs}} f(q_{fix} \mid H_0)\, dq_{fix} < p_{float} = \int_{t_{obs}} f(q_{float} \mid H_0)\, dq_{float}$$

$$trial\# = \frac{\int_{t_{obs}} f(q_{float} \mid H_0)\, dq_{float}}{\int_{t_{obs}} f(q_{fix} \mid H_0)\, dq_{fix}} = \frac{p_{float}}{p_{fix}}$$

Upcrossings

$$\hat{q}_0 \equiv q_0(\hat{m}) = -2\log\frac{\mathcal{L}(\mu=0)}{\mathcal{L}(\hat{\mu},\hat{m})} = \max_m[q_0(m)]$$

- Upcrossings: points where the $q_0(m)$ values become larger then some *u* are called *upcrossings*

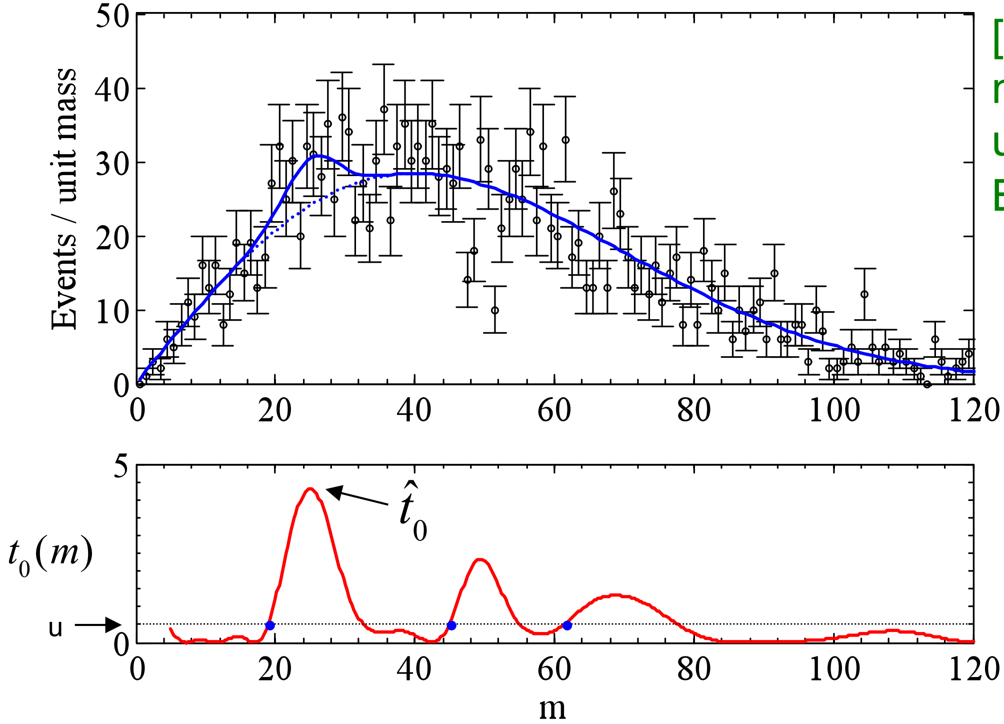


- The probability that the global maximum is above the level u is called *exceedance probability*
- *Excursion set, is the set of m with a probability above the level u*
- $p - value\ of\ \hat{q}_0 = \mathrm{P}\left(\max_m[q(m)] \geq q_{obs}\right)$

Trial factors for the look elsewhere effect in high energy physics

(arXiv:1005.1891)

Eilam Gross and Ofer Vitells Eur.Phys.J.C70:525-530,2010



[R.B. Davies, Hypothesis testing when a nuisance parameter is present only under the alternative. Biometrika **74**, 33–43 (1987)]

$$E[N_u] = \mathcal{N}_1 e^{-u/2}$$

The only unknown is $\mathcal{N}_1$ which can be estimated from the average number of upcrossings at some low reference level (with just a few MC tosses)

$$\mathcal{N}_1 \cong \left\langle N_{u_0} \right\rangle e^{u_0/2}$$

$$\mathrm{P}(\hat{q}_0 > u) \leq E[N_u] + \mathrm{P}(q_0(0) > u) = \mathcal{N}_1 e^{-u/2} + \frac{1}{2} P(\chi_1^2 > u)$$

The p-value can then be estimated by Davies' formula

Calculate the asymptotic trial factor

Trial factors for the look elsewhere effect in high energy physics

Eilam Gross and Ofer Vitells   Eur.Phys.J.C70:525-530,2010

- We show (arXiv:1005.1891) that the trial factor can be approximated by

$$traial\# \equiv \frac{p_{float}}{p_{fix}} \approx 1 + \sqrt{\frac{\pi}{2}} < N > Z_{fix}$$

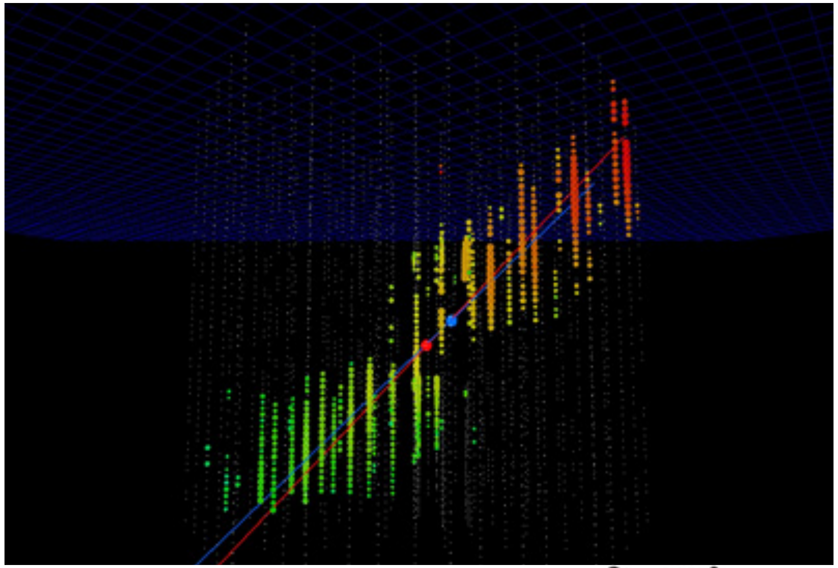
where $<N>=E[N_u]$ is the average number of upcrossings of $q=-2\ln L$ in the fit range (~Range/Resolution)

- We find

$$traial\# \sim \frac{Range}{Resolution} \cdot Z_{fix}$$

2-D example: IceCube search for astrophysical neutrino point sources

Estimating the significance of a signal in a multi-dimensional search.
Ofer Vitells & Eilam Gross, arXiv 1105.4355 submitted to Astroparticle Physics



IceCube looks for neutrino sources,

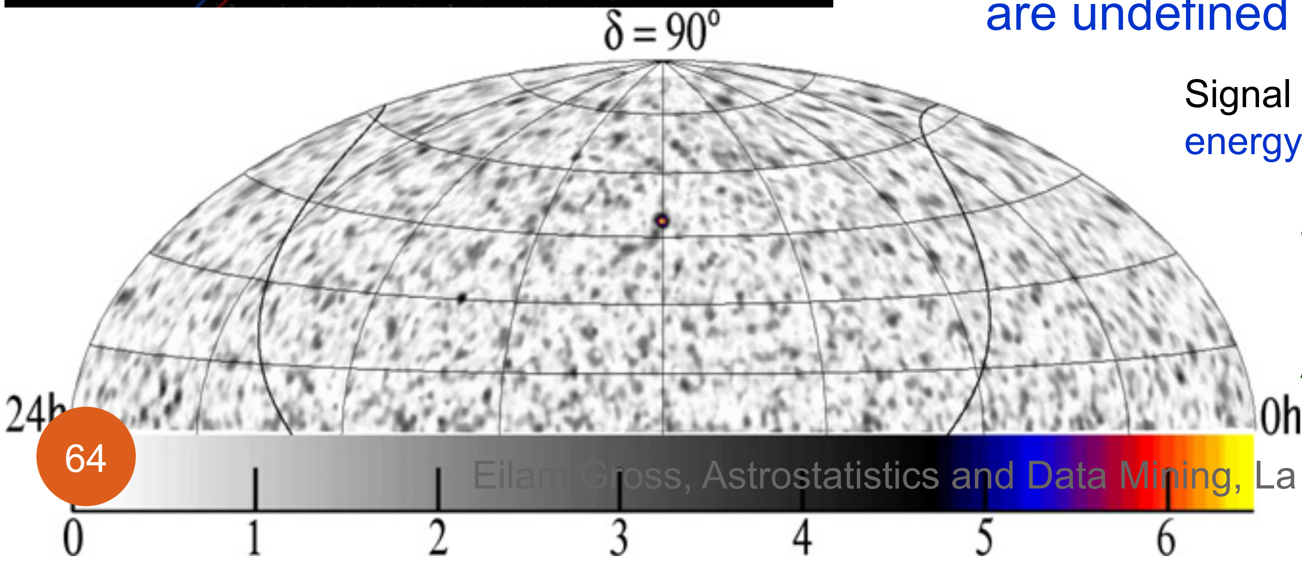
2-D Search over the sky (θ,φ),

Test statistics is denoted by q_0

$$q_0(n_s = 0) = -2ln\frac{L(n_s(\theta,\phi) = 0)}{L(\hat{n}_s(\hat{\theta},\hat{\phi}))}$$

Here there are 2 parameters which are undefined under the null hypothesis

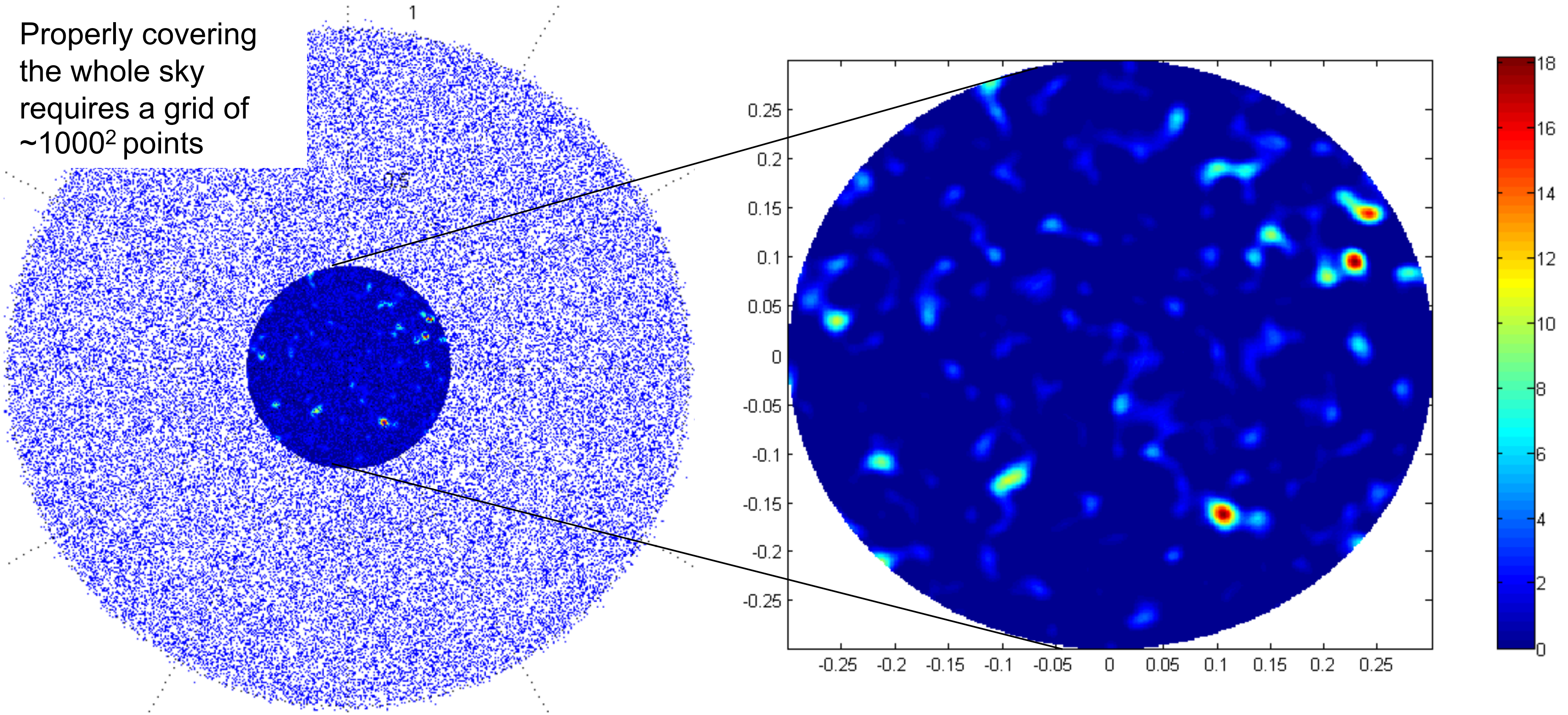
Signal parameters can also include energy and time, not considered here



J. Braun, J. Dumm, F. De Palma, C. Finley, A. Karle, and T. Montaruli, Astropart. Phys. 29, 299 (2008); [arXiv:0801.1604]

2-D example: search for neutrino sources (IceCube)

Properly covering the whole sky requires a grid of ~1000^2 points



IceCube simulated background data (1 year) 67,000 events, provided by Jim Braun & Teresa Montaruli

Significance map

$$q_0(\theta, \varphi)$$

The case of n signal nuisance parameters

- The upcrossings formula is a special case of a more general result which gives ***the expectation of the Euler characteristic of the excursion set*** of a random field over a general *n*-dimensional manifold

$$E[\varphi(A_u)] = \sum_{d=0}^{n} \mathcal{N}_d \rho_d(u)$$

- Here:

  A_u is the excursion set of the field above a level *u* (set of points where $q_0(\theta)>u$)

  $\varphi(A_u)$ is it's Euler characteristic

  ρ_d are 'universal' functions (depend only on the level *u* and the type of distribution)

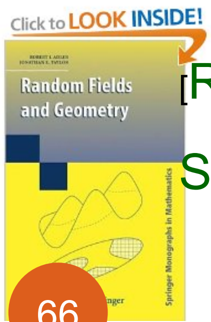
e.g. for a chi[2] field with *s* degrees of freedom:

$$\rho_0(u) = P(\chi_s^2 > u)$$

$$\rho_1(u) = u^{(s-1)/2} e^{-u/2}$$

$$\rho_2(u) = u^{(s-2)/2} e^{-u/2}[u-(s-1)]$$

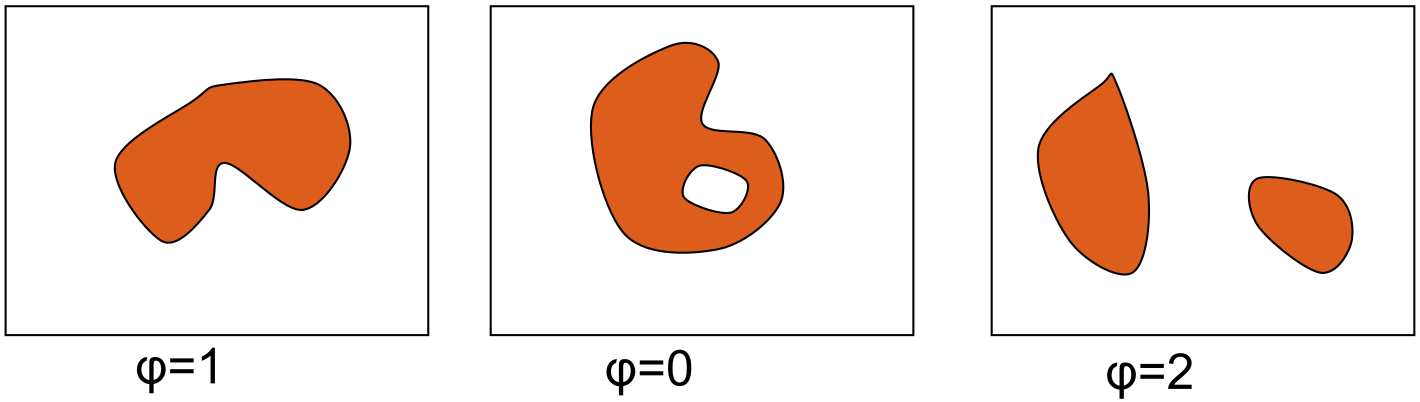
...



[R.J. Adler and J.E. Taylor, *Random Fields* (2007), Springer Monographs in Mathematics]

Euler characteristic

- Number of disconnected components minus number of `holes'



WIKIPEDIA
The Free Encyclopedia

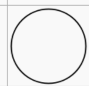
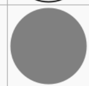
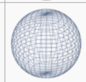
Euler characteristic

From Wikipedia, the free encyclopedia

In mathematics, and more specifically in algebraic topology and polyhedral combinatorics, the **Euler characteristic** (or **Euler–Poincaré characteristic**) is a topological invariant, a number that describes a topological space's shape or structure regardless of the way it is bent. It is commonly denoted by χ (Greek letter chi).

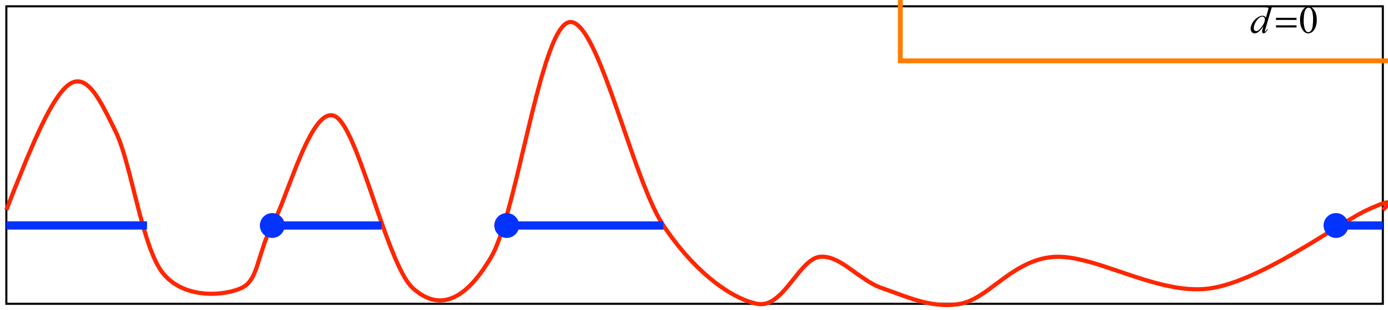
The Euler characteristic was originally defined for polyhedra and used to prove various theorems about them, including the classification of the Platonic solids. Leonhard Euler, for whom the concept is named, was responsible for much of this early work. In modern mathematics, the Euler characteristic arises from homology and connects to many other invariants.

67

Name	Image	Euler characteristic
Interval		1
Circle		0
Disk		1
Sphere		2
Torus (Product of two circles)		0
Double torus		−2

Euler characteristic

$$E[\varphi(A_u)] = \sum_{d=0}^{n} \mathcal{N}_d \rho_d(u)$$



In 1 dimension with

1 paramter of interest

1 signal nuisance parameter

$$\varphi(A_u) = N_u + \mathbf{1}_{[q_0(0)>u]}$$

$$E[\varphi(A_u)] = E[N_u] + \mathrm{P}(q_0(0) > u)$$
$$= \mathcal{N}_1 e^{-u/2} + \frac{1}{2} P(\chi_1^2 > u)$$

(Davies' Bound)

The general n-D case,

s parameters of interest,

n signal nuisance parameters

$\mathcal{N}_0 =$ Euler characteristic of the n-D manifold

$$\rho_0(u) = \mathrm{P}(\chi_s^2 > u)$$

$$\rho_1(u) = u^{(s-1)/2} e^{-u/2}$$

$$\rho_2(u) = u^{(s-2)/2} e^{-u/2}[u - (s-1)]$$

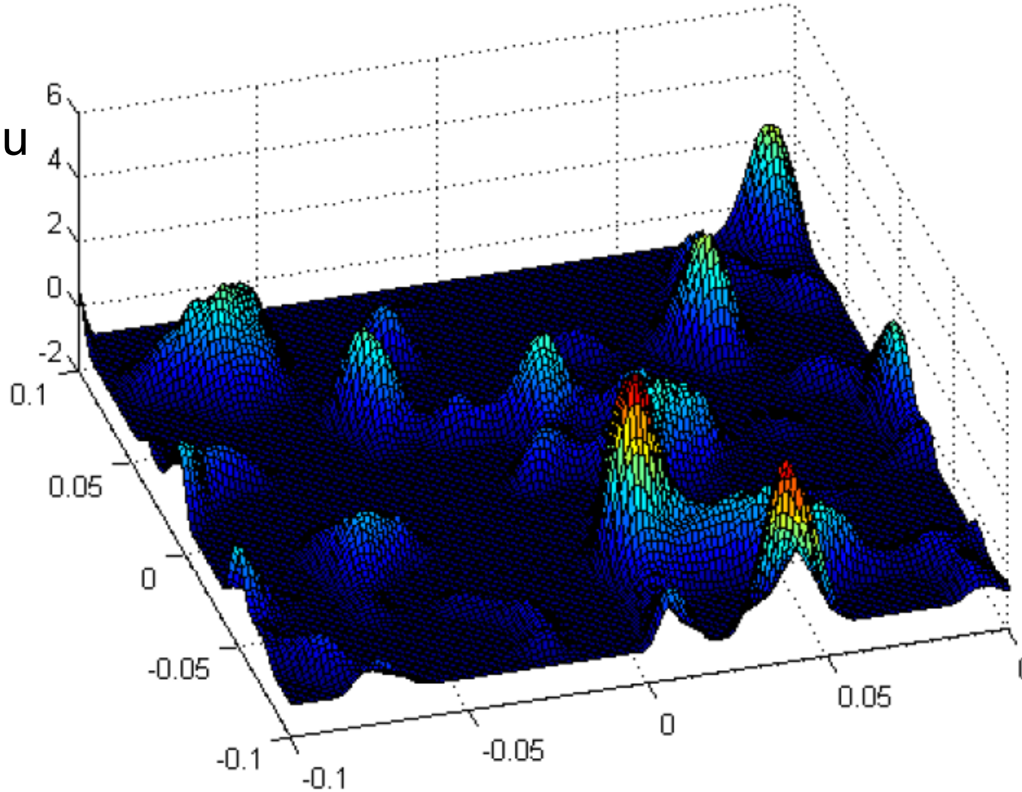
...

In general for high-level excursions $E[\varphi(A_u)] \to \mathrm{P}(\max_\theta [q_0(\theta)] \geq u)$

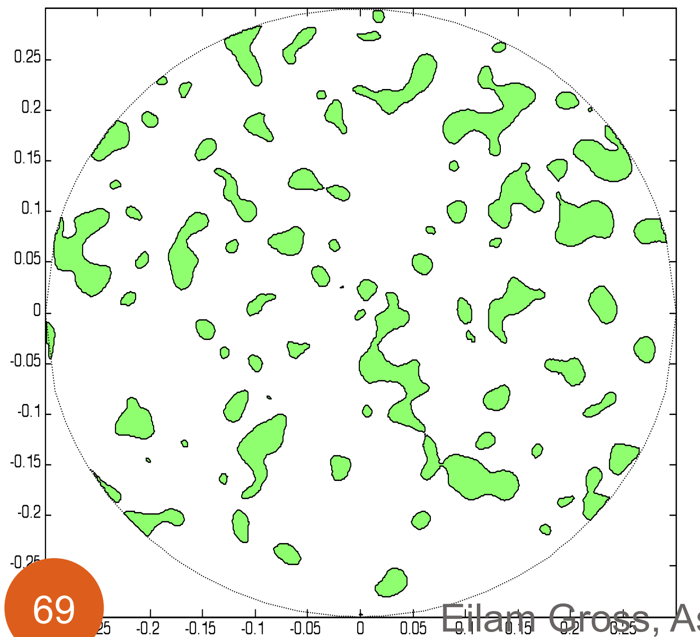
(When $E[\varphi(A_u)] \ll 1$)

[J. Taylor, A. Takemura, and R. J. Adler, Ann. Probab. 33, 4 (2005)]

To find, say, prob(max[q_0]>30)
All there is to do is
cut at two low values of u
say u=0,1
and find the Euler characteristic

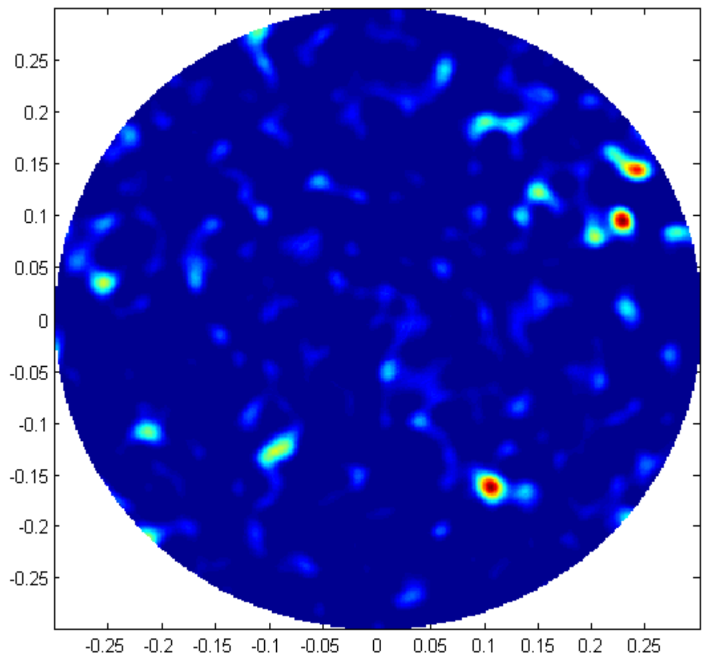


Excursion set
(u=1)

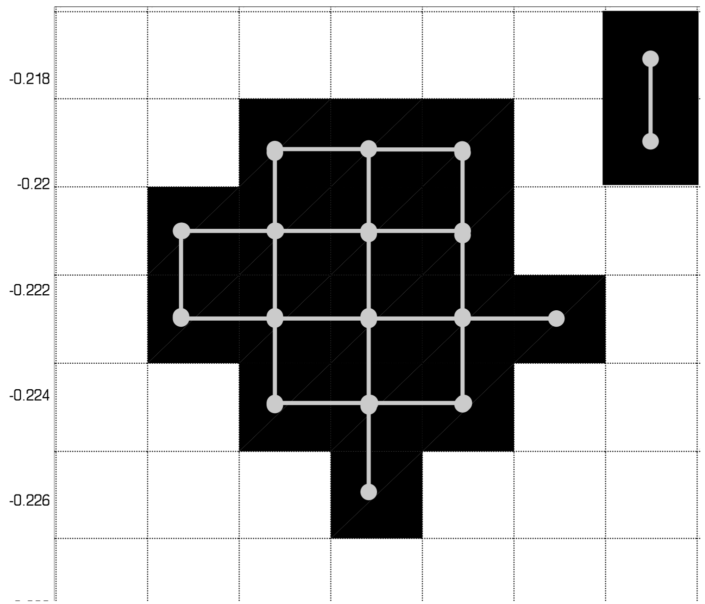


$q_0(\theta,\varphi)$

Significance map



Calculation of the Euler characteristic



- Usually we have $q(\theta)$ calculated on a grid of points
- Calculation of the E.C. is straightforward:
- φ = #points - #edges + #faces
- Generalizes to higher dimensions

φ = 18(points) – 23 (edges) + 7(faces)

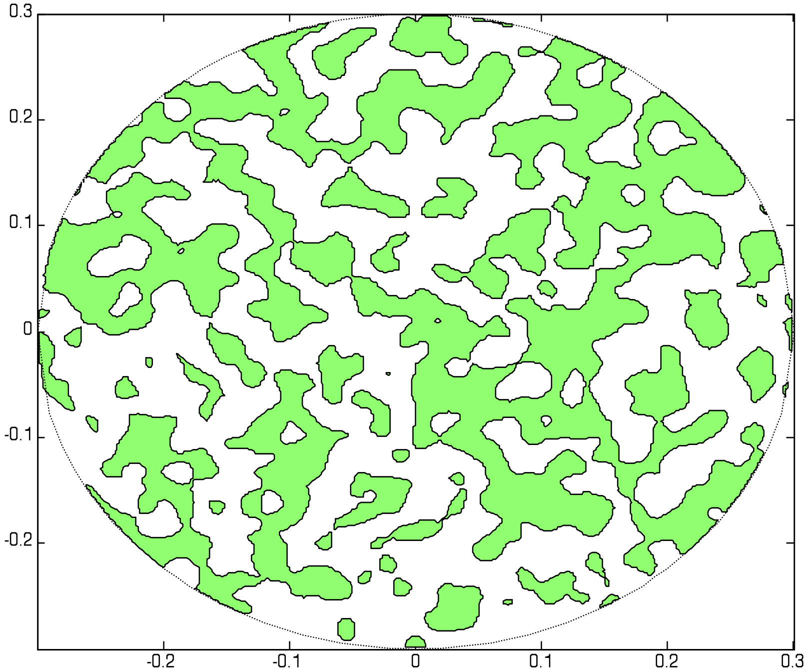
= 2

2-d example: search for neutrino sources (IceCube)

For a chi[2] field in 2 dimensions (n=2,s=1):

$$E[\varphi(A_u)] = \sum_{d=0}^{n} \mathcal{N}_d \rho_d(u)$$

$$\rho_0(u) = P(\chi_s^2 > u)$$
$$\rho_1(u) = u^{(s-1)/2} e^{-u/2}$$
$$\rho_2(u) = u^{(s-2)/2} e^{-u/2}[u-(s-1)]$$
$$\ldots$$



$$\varphi(A_u)] = \frac{1}{2} P(\chi^2 > u) + (\mathcal{N}_1 + \mathcal{N}_2 \sqrt{u}) e^{-u/2}$$

Estimating the significance of a signal in a multi-dimensional search.
Ofer Vitells & Eilam Gross, arXiv 1105.4355 submitted to Astroparticle Physics

2-d example: search for neutrino sources (IceCube)

For a chi² field in 2 dimensions:

$$E[\varphi(A_u)] = \frac{1}{2}\mathrm{P}(\chi^2 > u) + (\mathcal{N}_1 + \mathcal{N}_2\sqrt{u})e^{-u/2}$$

Estimate E[φ] at two levels, e.g. 0 and 1, and solve for $\mathcal{N}_1$ and $\mathcal{N}_2$

From 20 bkg. Simulations:

$$\langle\varphi_0\rangle = 33.5 \pm 2$$

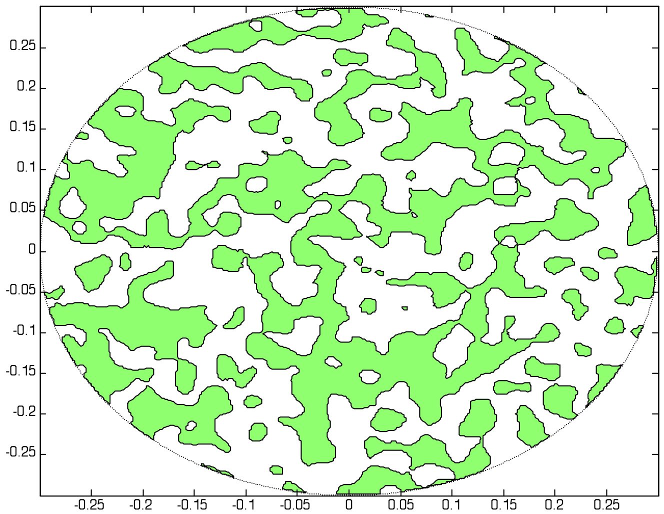
$$\langle\varphi_1\rangle = 94.6 \pm 1.3$$

$$\Downarrow$$

$$\mathcal{N}_1 = 33 \pm 2$$

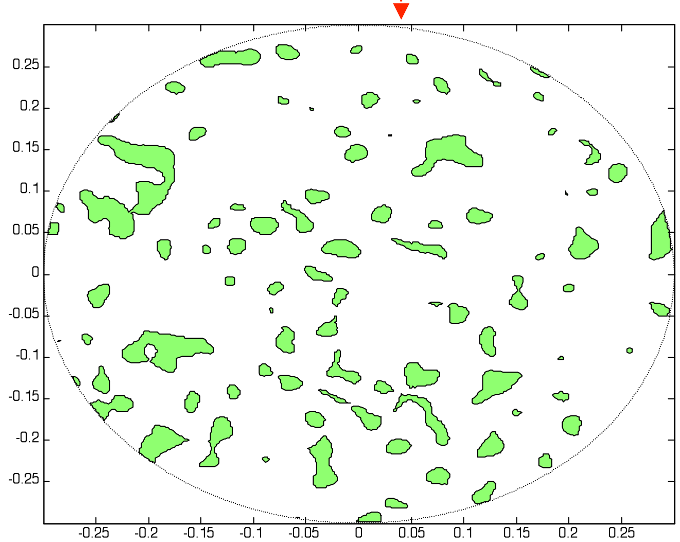
$$\mathcal{N}_2 = 123 \pm 3$$

u=0



φ=35

u=1



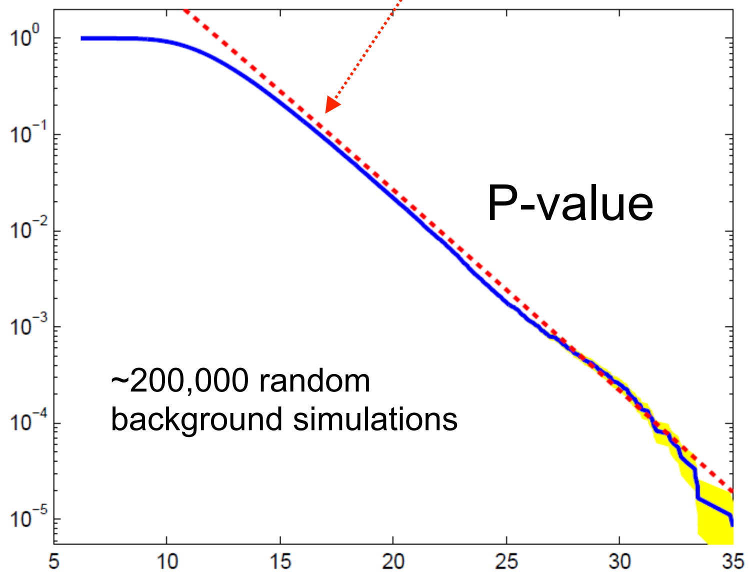
φ=95

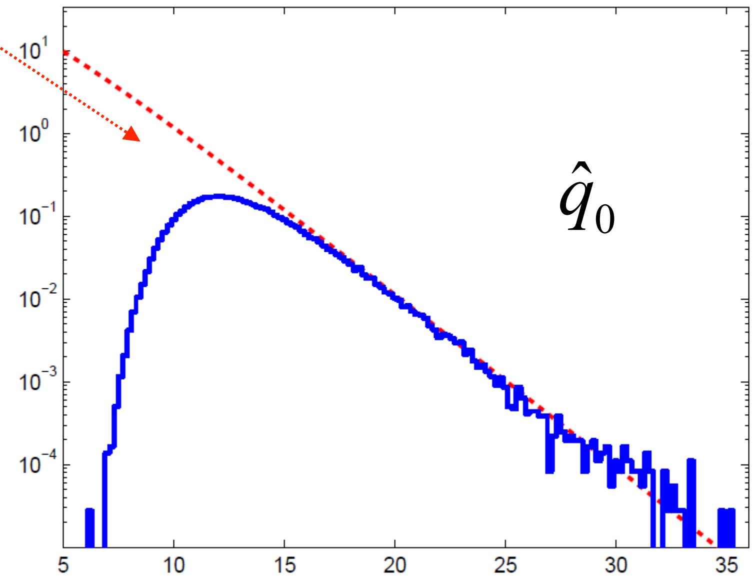
Estimating the significance of a signal in a multi-dimensional search.
Ofer Vitells & Eilam Gross, arXiv 1105.4355 submitted to Astroparticle Physics

2-d example: search for neutrino sources (IceCube)

$$E[\varphi(A_u)] = \frac{1}{2}\mathrm{P}(\chi^2 > u) + (\mathcal{N}_1 + \mathcal{N}_2\sqrt{u})e^{-u/2}$$

$$\mathcal{N}_1 = 33 \pm 2$$
$$\mathcal{N}_2 = 123 \pm 3$$



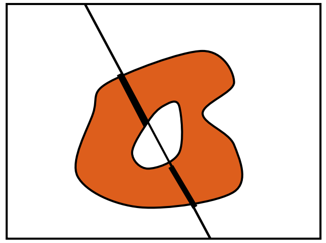


e.g.: P(max q_0>30) = $(2.5 \pm 0.4)\times10^{-4}$ (estimated)
E.C. Formula : $(2.28 \pm 0.06)\times10^{-4}$

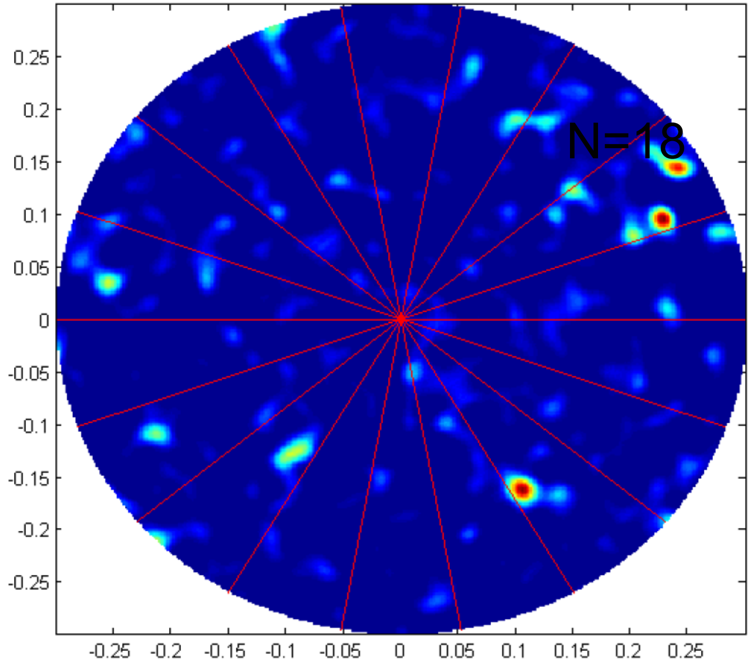
Estimating the significance of a signal in a multi-dimensional search.
Ofer Vitells & Eilam Gross, arXiv 1105.4355 submitted to Astroparticle Physics

Slicing

Estimating the significance of a signal in a multi-dimensional search.
Ofer Vitells & Eilam Gross, arXiv 1105.4355
submitted to Astroparticle Physics



φ=0=1+1-2



- Exploit the azimuthal angle symmetry to reduce computations:

$$\varphi(A \cup B) = \varphi(A) + \varphi(B) - \varphi(A \cap B)$$

Divide to N slices:

$$\varphi = \sum_i [\varphi(\text{slice}_i) - \varphi(\text{edge}_i)] + \varphi(0)$$

$$E[\varphi] = N \times (E[\varphi(\text{slice})] - E[\varphi(\text{edge})]) + E[\varphi(0)]$$

40 “slice” simulations

$$E[\varphi(\text{slice})] = ((6 \pm 0.5) + (6.7 \pm 0.8)\sqrt{u})e^{-u/2}$$

$$E[\varphi(\text{edge})] = (4.4 \pm 0.2)e^{-u/2}$$

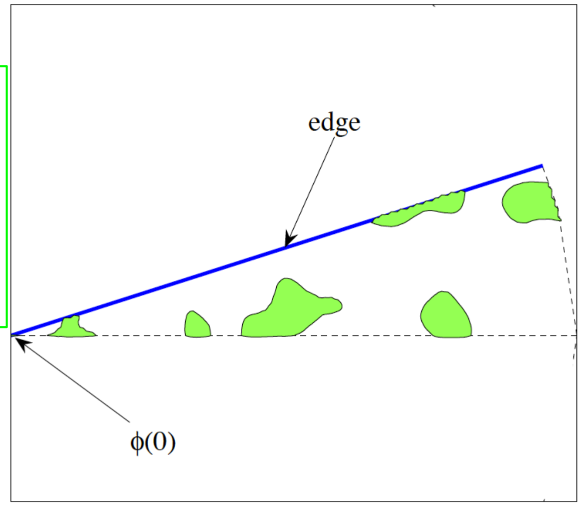
⇩

$$\mathcal{N}_1 = 28 \pm 9$$

Consistent with full sky

$$\mathcal{N}_2 = 120 \pm 14$$

In this example
φ(0)=0
φ(edge)=2
Φ(slice)=6



EXCLUSION

Castling the Hypotheses

NULL

H_0- SM w/o Higgs

ALTERNATE

H_1- SM with Higgs

- Reject H_0 in favor of H_1 – A DISCOVERY

$$q_0^{PL} = -2\ln \frac{L\left(b(\hat{\hat{\theta}}_b)\right)}{L\left(\hat{\mu}s + b(\hat{\theta})\right)}$$

$$\hat{\hat{\theta}}_b : MLE\ of\ \ L(b(\theta));\quad \hat{\mu},\hat{\theta} : MLE\ of\ \ L(\mu s + b(\theta))$$

Castling the Hypotheses

NULL

H_1 - SM with Higgs

ALTERNATE

H_0 - SM w/o Higgs

- Reject H_1 in favor of H_0 – Excluding H_1

$$q_0^{PL} = -2\ln \frac{L\left(b(\hat{\hat{\theta}}_b)\right)}{L\left(\hat{\mu}s + b(\hat{\theta})\right)} \quad \Rightarrow \quad q_1^{PL} = -2\ln \frac{L\left(s + b(\hat{\hat{\theta}}_{s+b})\right)}{L\left(\hat{\mu}s + b(\hat{\theta})\right)}$$

$$\hat{\hat{\theta}}_b, \hat{\hat{\theta}}_{s+b} : MLE\ of\ \ L(b(\theta)), L(s+b(\theta)); \quad \hat{\mu}, \hat{\theta} : MLE\ of\ \ L(\mu s + b(\theta))$$

Exclusion

- Test the H_μ hypothesis, $\langle n \rangle = \mu s(m_H) + b$
- $\mu = \frac{\sigma}{\sigma_{SM}}$ is the signal strength which is a parameter of interest.

$$q_\mu^{PL} = -2\ln \frac{L\left(\mu s + b(\hat{\hat{\theta}}_{s+b})\right)}{L\left(\hat{\mu} s + b(\hat{\theta})\right)}$$

$$\hat{\hat{\theta}}_{s+b} : MLE\ of\ L(s+b(\theta));\quad \hat{\mu}, \hat{\theta} : MLE\ of\ L(\mu s + b(\theta))$$

Wilks Theorem

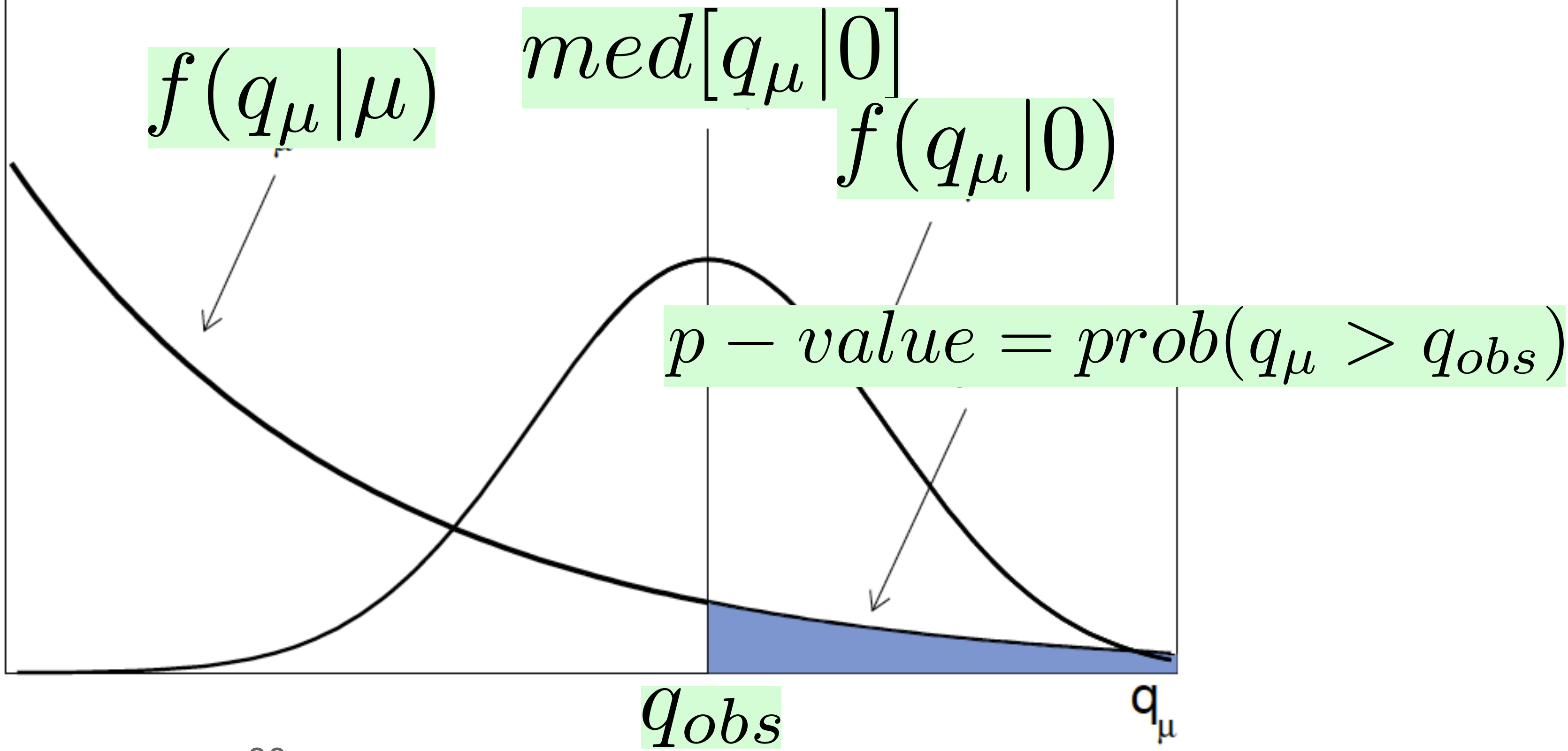
- We use the Profile Likelihood

$$q_\mu = -2\log\frac{\max_\theta L(\mu,\theta)}{\max_{\mu,\theta} L(\mu,\theta)} = -2\log\frac{L(\mu,\hat{\hat{\theta}}_\mu)}{L(\hat{\mu},\hat{\theta})}$$

- The pdf $f(q_\mu | H_\mu)$ is asymptotically not dependent on the nuisance parameters, that's a big advantage of the PL
- (Note, we test the signal hypothesis (H_μ) trying to reject it in order to establish an exclusion at the 95% CL.)

Exclusion sensitivity

- Here you want to find the sensitivity of the experiment to exclude some signal s with a strength μ



Exclusion

- Test the H_μ hypothesis, $<n>=\mu s(m_H)+b$
- $\mu = \frac{\sigma}{\sigma_{SM}}$ is the signal strength which is a parameter of interest.

$$q_\mu^{PL} = -2\ln \frac{L\left(\mu s + b(\hat{\hat{\theta}}_{s+b})\right)}{L\left(\hat{\mu} s + b(\hat{\theta})\right)}$$

- By testing the signal hypothesis (H_μ) we can construct a *95%* confidence (frequentist) interval

  CI: $[0, \mu_{95}]$ (CI: Confidence Interval for $\mu = \frac{\sigma}{\sigma_{SM}}$)

- If $\mu_{95}<1$ the SM Higgs (H_1) is excluded at the *95%* CL.

  A SUSY Higgs (with a smaller signal strength) can still be hidden there…

Exclusion

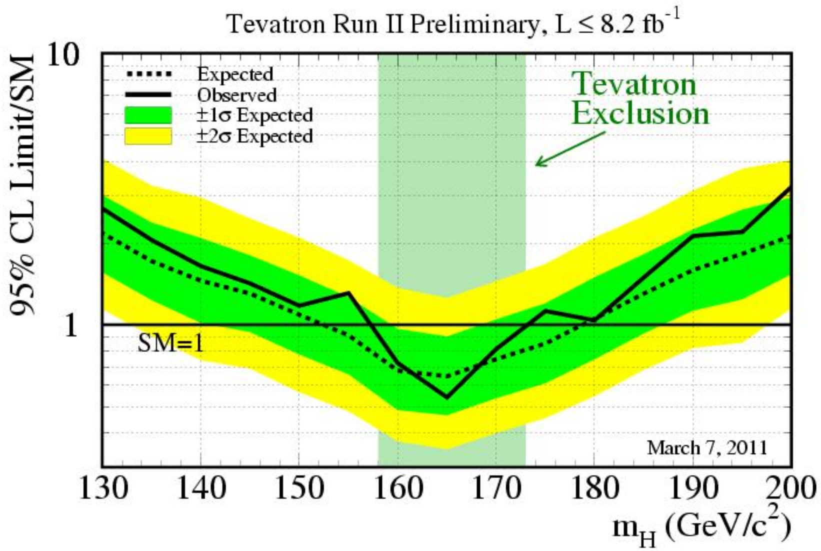
- Test the H_μ hypothesis, $\langle n \rangle = \mu s(m_H) + b$
- $\mu = \frac{\sigma}{\sigma_{SM}}$ is the signal strength which is a parameter of interest.

$$q_\mu^{PL} = -2\ln \frac{L\left(\mu s + b(\hat{\hat{\theta}}_{s+b})\right)}{L\left(\hat{\mu} s + b(\hat{\theta})\right)}$$

- By testing the signal hypothesis (H_μ) we can construct a *95%* confidence (frequentist) or credibility (Bayesian) interval CI: [*0*, μ_{95}] (CI: Confidence or (

- If $\mu_{95} < 1$ the SM Higgs (H_1) is excluded at the *95%* CL.

A SUSY Higgs (with a smaller signal strength) can still be hidden there…



Combined CDF and D0 Upper Limits on Standard Model Higgs-Boson Production
arXiv:0901.2427 [hep-ph]
arXiv:0811.3458 [hep-ph]

The Euivalence of CL and p-value

- Test the H_μ $(\mu s(m_H)+b)$ hypothesis
- Find the p-value under H_μ $p_\mu(m_H)=\int_{q_\mu,obs}^{\infty} f(q_\mu \mid H_\mu)\,dq_\mu$
- If $p_\mu(m_H)<5\%$ the H_μ hypothesis is rejected
- Find $\mu 95(m_H)$ such that $p_{\mu 95}(m_H)=5\%$
- Had the signal existed, 95% of the intervals $[0, \mu 95(m_H)]$ would contain its true strength, $\mu(m_H)<\mu 95(m_H)$
- $\mu 95(m_H)$ is an upper bound on $\mu(m_H)$ @ 95% CL
- If $\mu 95(m_H)<1$, a SM Higgs with a mass m_H

  is excluded at >95% CL→ $p_\mu=1\text{-}CL$

- *CL is the s+b CL hence CL=CLs+b*

Asymptotic formulae for likelihood-based tests of new physics

Cowan, Cranmer, Gross, Vitells, arXiv:1007.1727,
EPJC 71 (2011) 1-19

Significance test using profile likelihood ratio

Systematics included via nuisance parameters

Distributions in large sample limit, no MC used.

http://arxiv.org/abs/1007.1727v2

The one-sided test statistics

http://arxiv.org/abs/1007.1727v2 (Cowan,,Cranmer,E.G.Vitells)

$$q_\mu = \begin{cases} -2\ln\dfrac{L(\mu,\hat{\hat{\theta}}_\mu)}{L(\hat{\mu},\hat{\theta})}; & \hat{\mu} < \mu \\ 0 & \hat{\mu} > \mu \end{cases}$$

$$p_\mu = \int_{q_{\mu,obs}}^{\infty} f(q_\mu \mid \mu)\, dq_\mu$$

- Wilks $f(q_\mu \mid \mu) \sim \frac{1}{2}\chi^2$

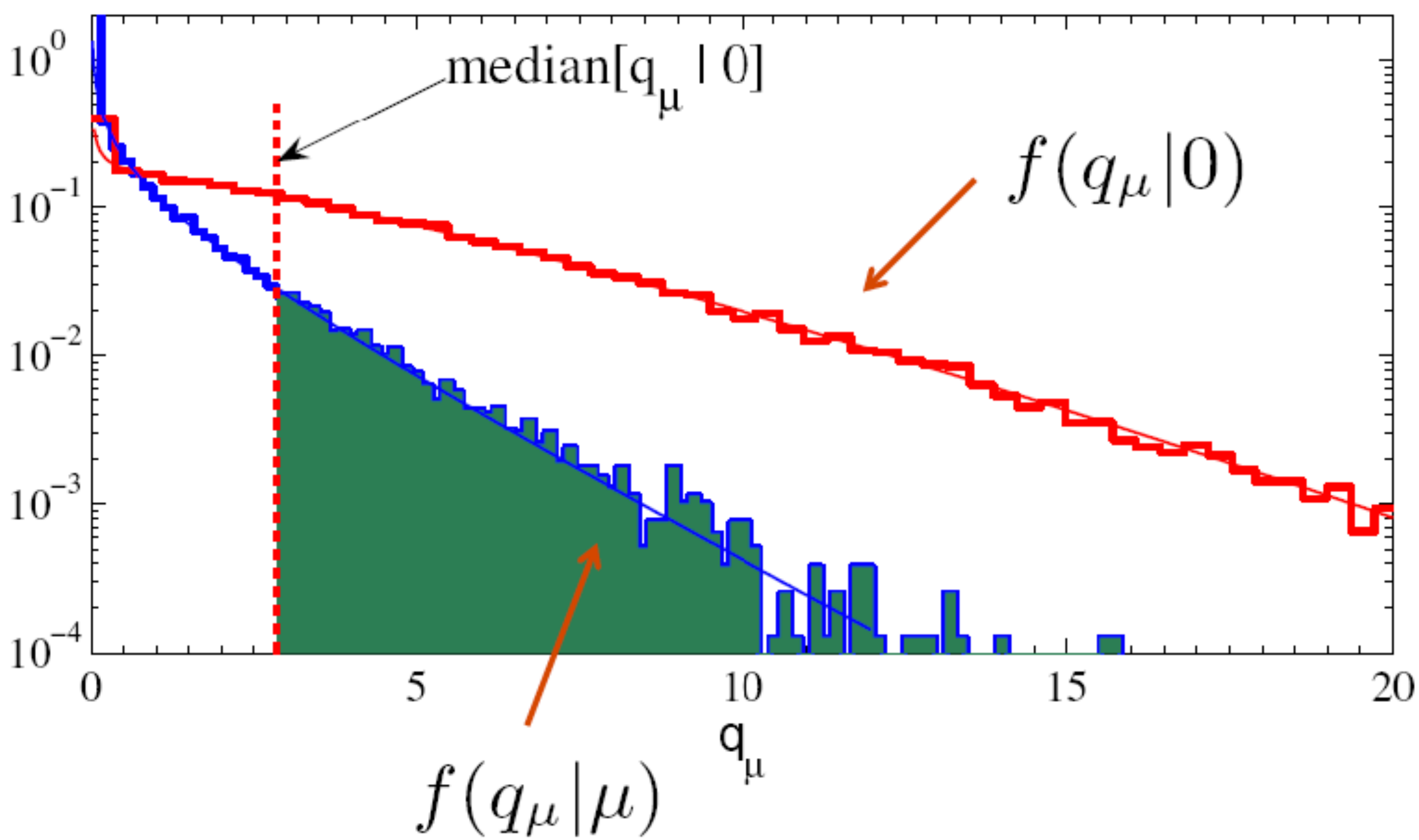
$$f(q_\mu|\mu) = \frac{1}{2}\delta(q_\mu) + \frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{q_\mu}}e^{-q_\mu/2}$$

- Wald $f(q_\mu \mid \mu') \sim \chi^2\left(\Lambda = \frac{(\mu-\mu')^2}{\sigma^2}\right)$

$$f(q_\mu|\mu') = \Phi\left(\frac{\mu'-\mu}{\sigma}\right)\delta(q_\mu) + \frac{1}{2}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{q_\mu}}\exp\left[-\frac{1}{2}\left(\sqrt{q_\mu} - \frac{(\mu-\mu')}{\sigma}\right)^2\right]$$

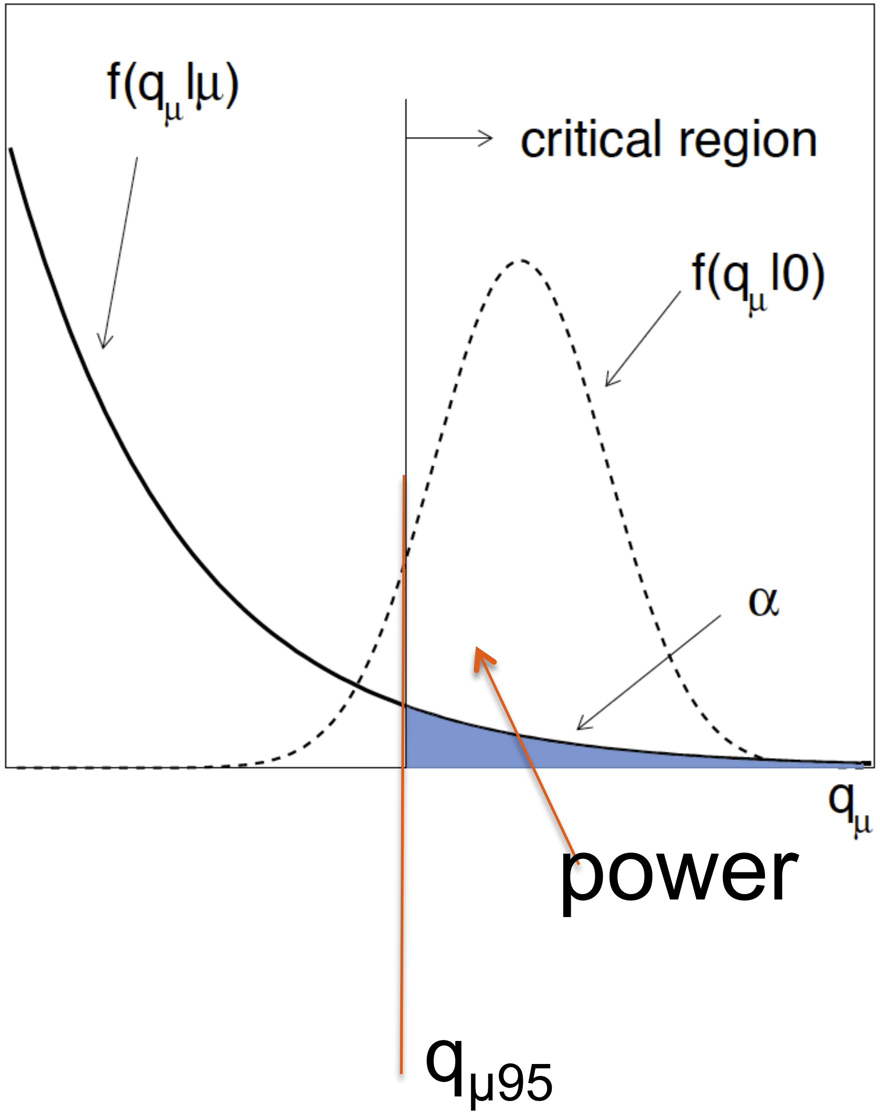
Test of Asymptotic formulae

Distributions of q_μ here for μ that gave $p_\mu = 0.05$. $\sqrt{q_{\mu 95}} = 1.64^2$



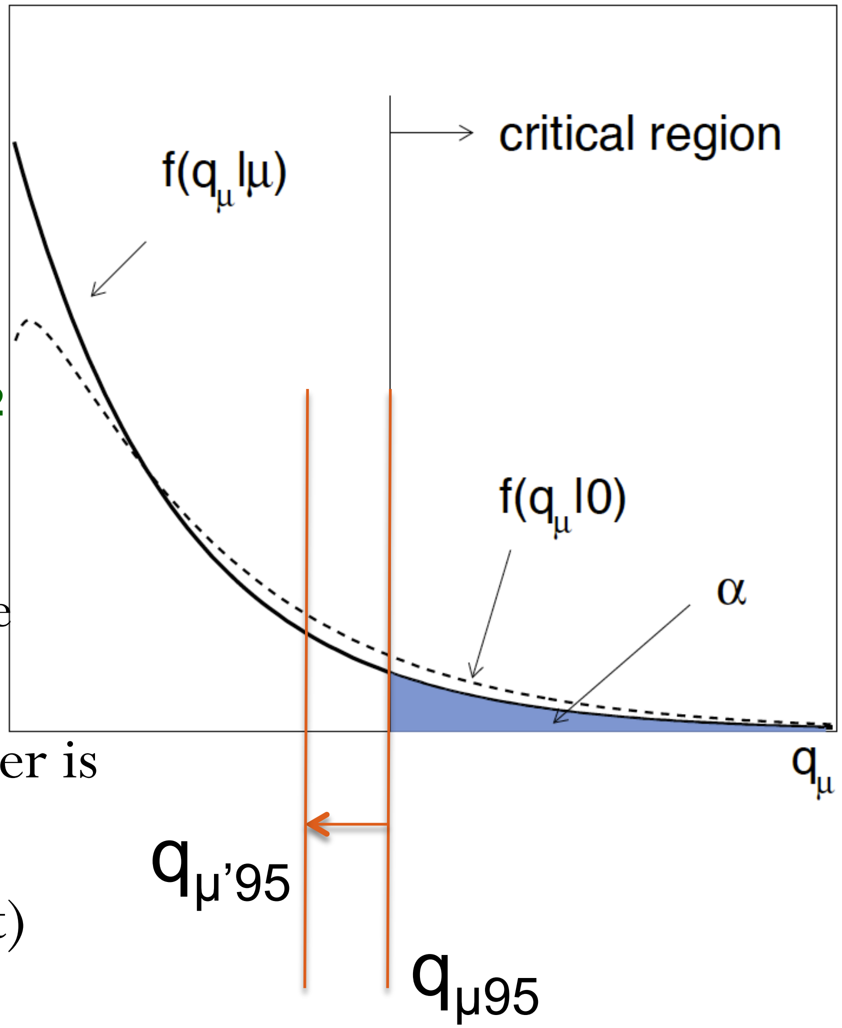
Statistical tests and Power

- Define a critical region α, say $\alpha = 5\%$
- If the observation $q_{\mu,obs}$ is in this region, the H_μ hypothesis is rejecte at the 95% CL
- The edge of the critical region is $q_{\mu 95}$ or $q_{\mu,up}$, (there $p_\mu = \alpha$)
- At the edge, $\alpha \Leftrightarrow$ CLsb
- Prob (reject $H_\mu \mid H_\mu$)=5%
- Power of the test
  Power = Prob ($H_0 \mid$ reject H_μ)
- If we solve for $q_{\mu,up} = q_{\mu,obs}$ then CLb $\Leftrightarrow$ power



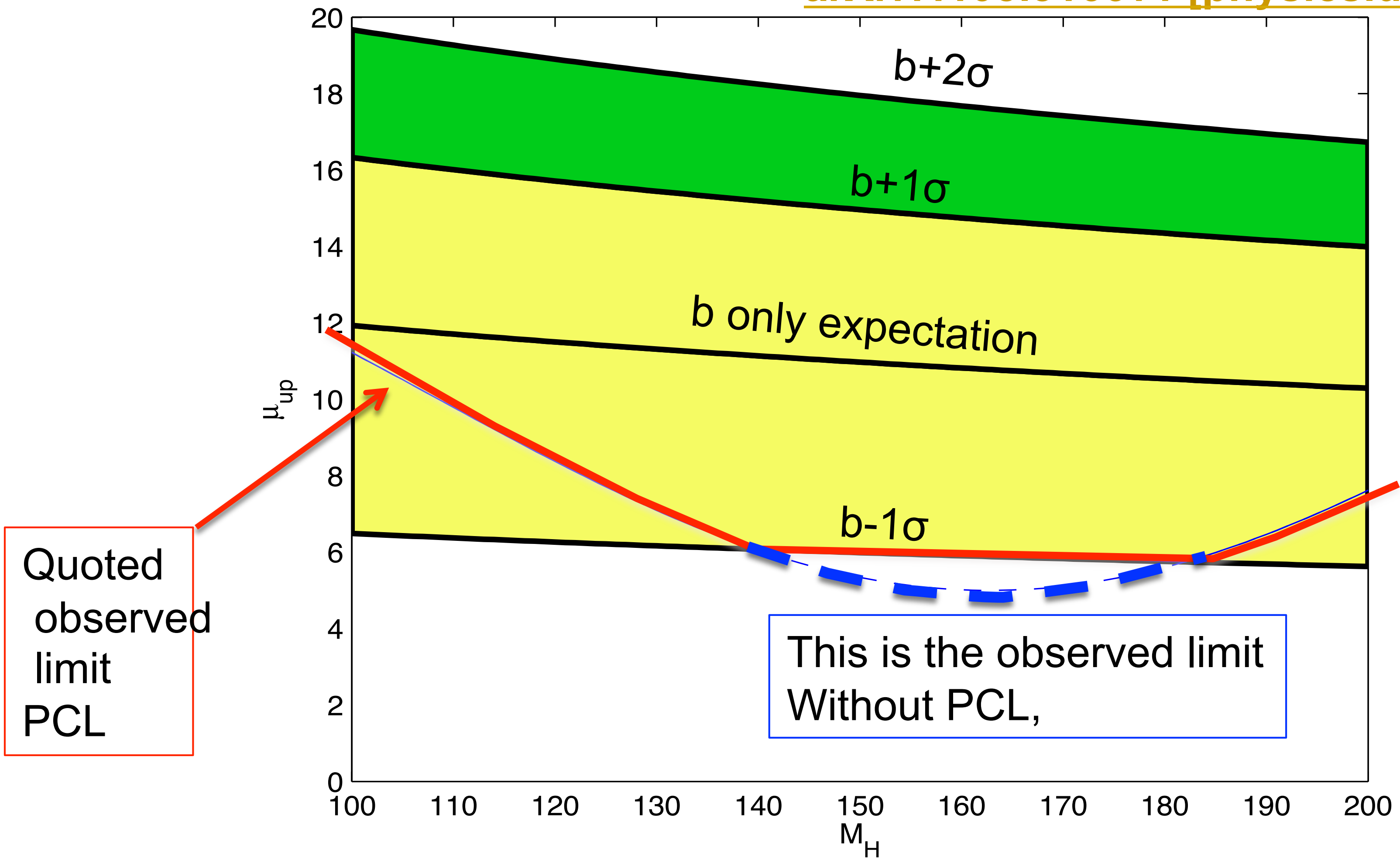
Statistical tests and Power

- Here the two hypotheses (signal and BG-only) can hardly be separated, s+b~b=>s~0
- The power is small
- You might exclude s~0
- To protect it:
  - CLs=CLsb/CLb Alex Read **J.Phys.G28:2693-2704,2002**
  - At the edge of the critical region $CLs=p'_{\mu}=\alpha/power>\alpha=p_{\mu}$
  - A new μ up is defined correposnds to the new $p'_{\mu}=CLs \Rightarrow q_{\mu' up}<q_{\mu\, up}$
- The PCL: redefine μ up so that the power is no less than 16%
- In both cases μ 'up> μ up (weaker limit)



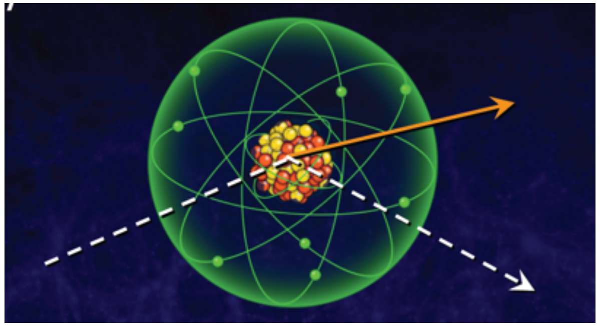
Power Constrained Limits

Cowan,Cranmer,E.G.,Vitells
arXiv:1105.3166v1 [physics.data-an]

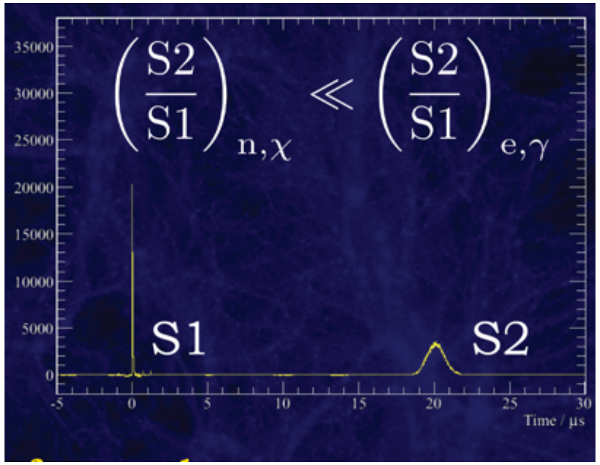


WIMPS Detection with XENON

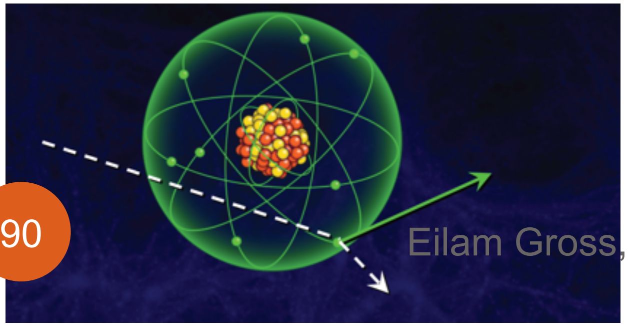
R. Lang

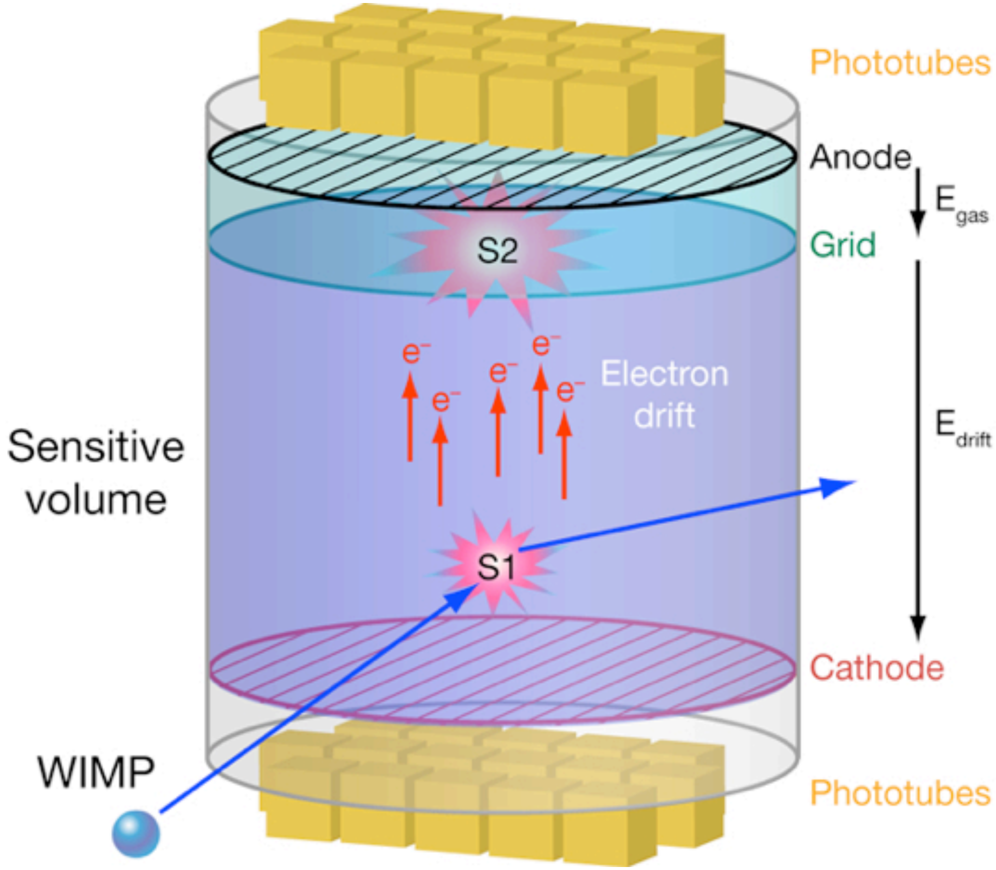


Nuclear Recoils



Electronic Recoils e/γ

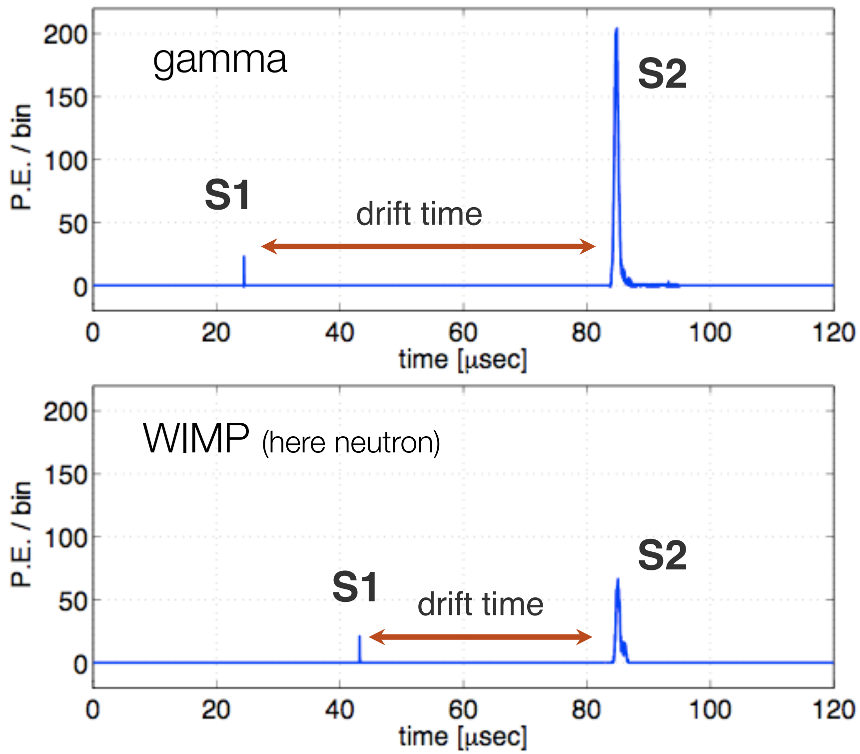


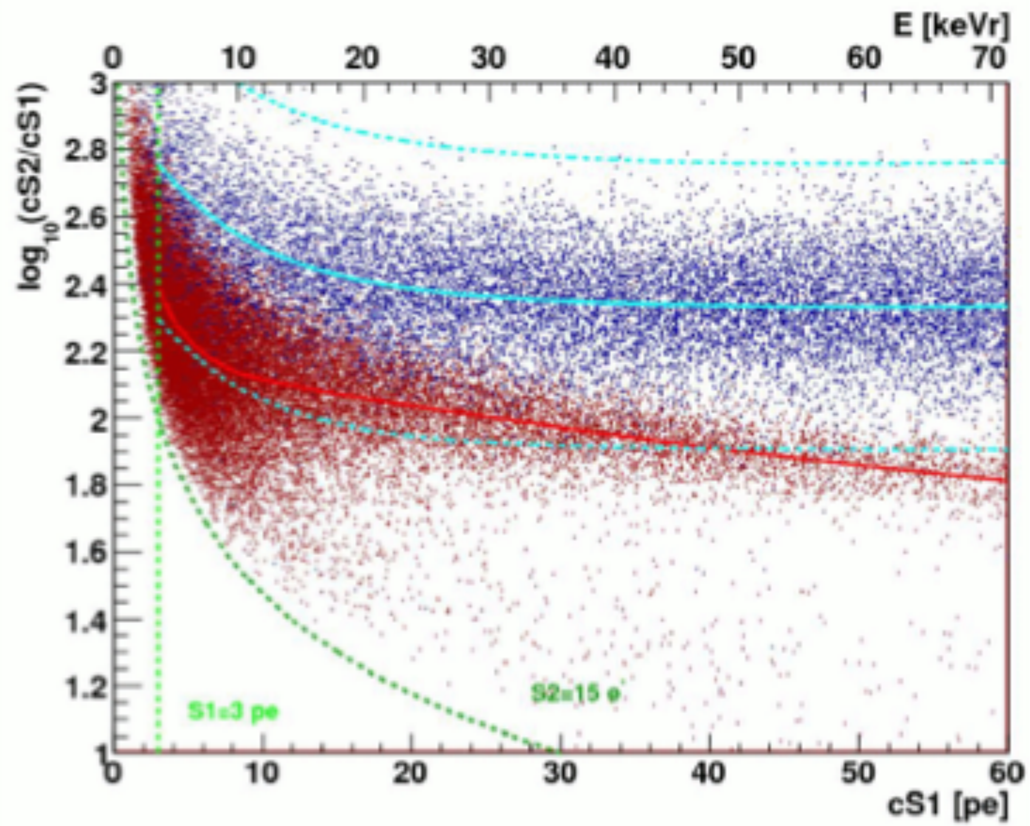


Leff is crucial, it sets the scale between S1 and the recoil energy

ER/NR Discrimination

- In this plot ^{60}Co (blue) and AmBe (red)
- ER/NR discrimination via S2/S1 ratio

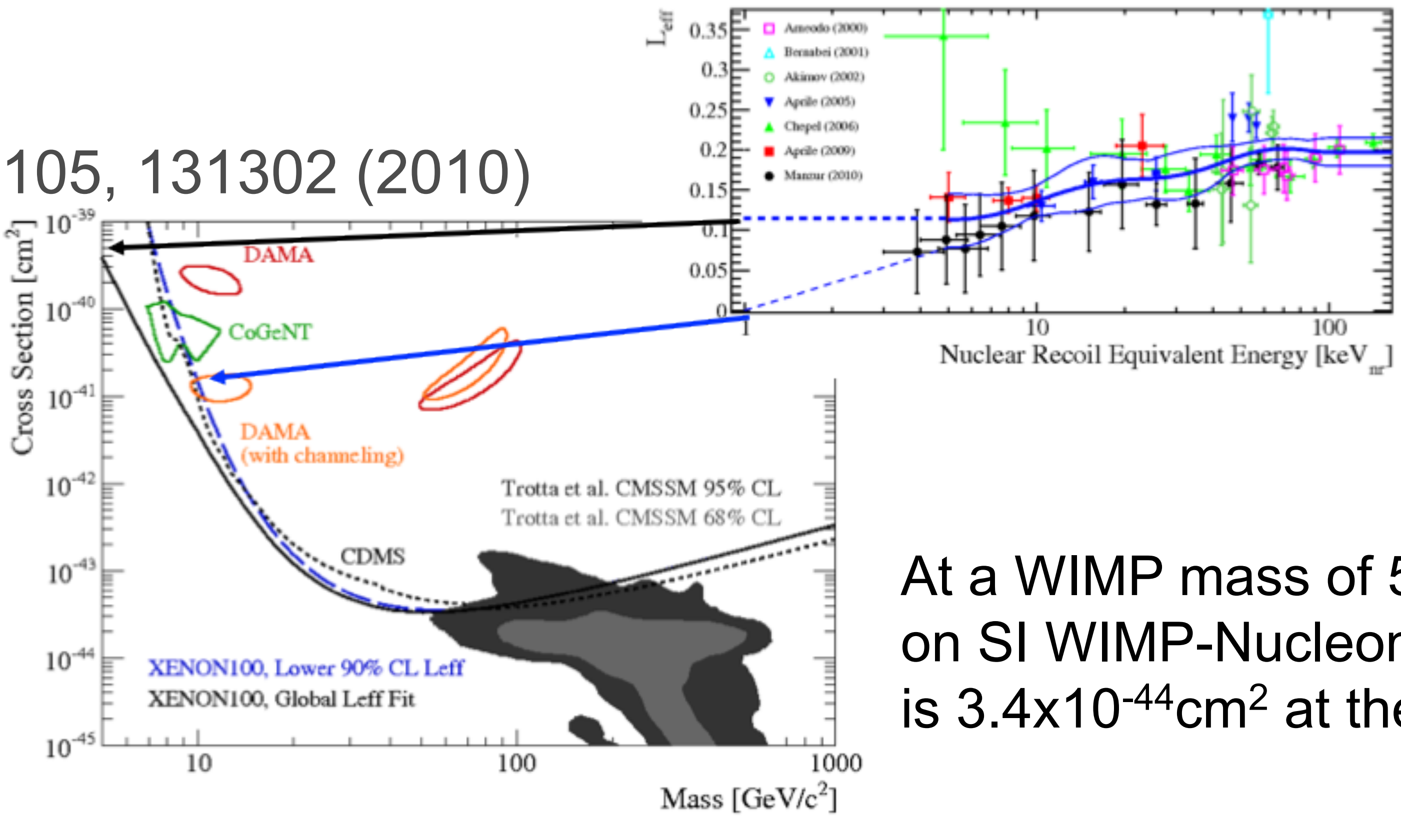




First published results of XENON100

- Observing no events in the signal region, the limit on the WIMP mass was obtained, based on an expectation of 2.3 signal events in the signal region (90% CL)
- The Leff uncertainty was taken into account by deriving two limit curves, one tighter than the other.

PRL 105, 131302 (2010)

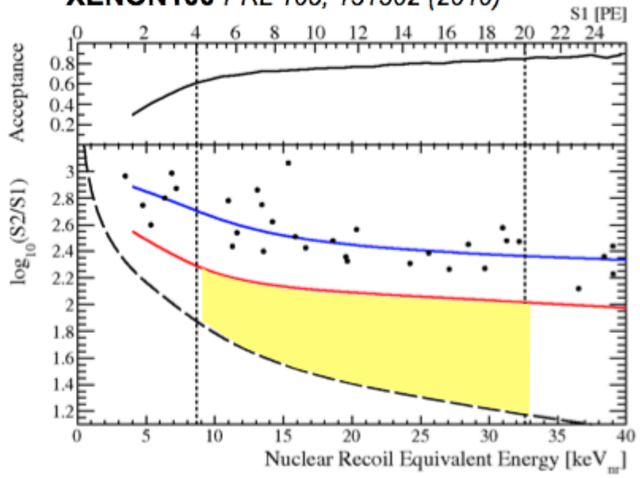


At a WIMP mass of 55 GeV the limit on SI WIMP-Nucleon cross section is $3.4 \times 10^{-44} cm^2$ at the 90% CL

A Likelihood Approach to XENON100 Data

- The traditional method for analyzing Dark Matter data defines a signal region via a hard cut
- Such a hard cut introduces sensitivity reduction and boundaries uncertainties

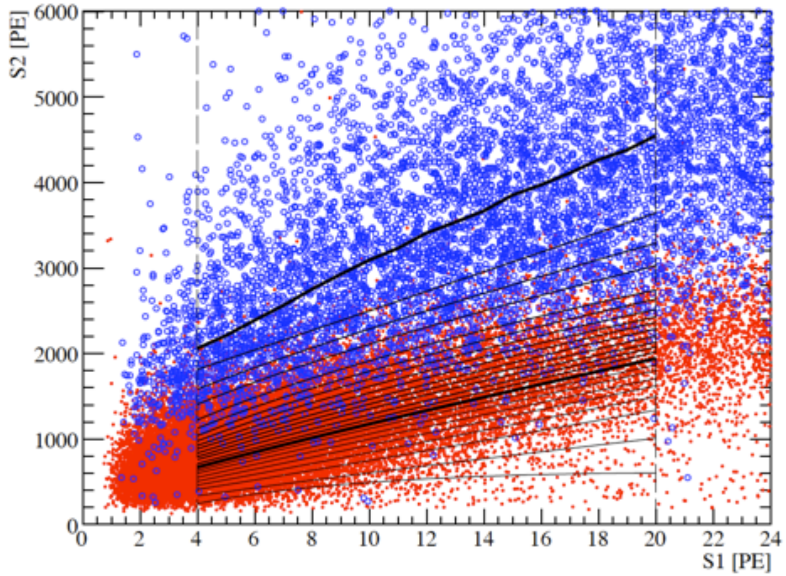
XENON100 *PRL 105, 131302 (2010)*

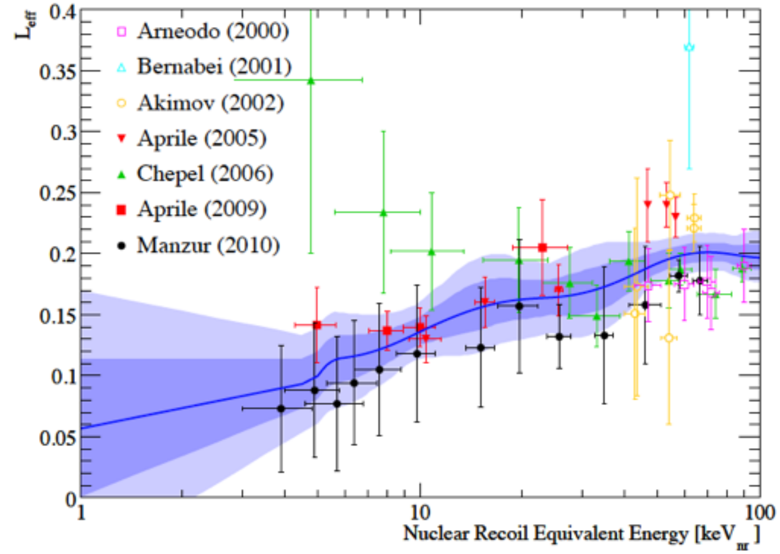


- In the traditional method uncertainties are not embedded in the method, leading sometimes to ambiguities (Leff)
- The traditional methods do not make full use of background information that exist (via calibration, MC or theory)
- Basing the analysis on zero observed events (e.g. maximum gap) does not enable a discovery.
  We are not in this "game" for exclusion.

The Profile Likelihood Approach

- The basic idea is to use existing measurements such as ^{60}Co gamma calibration and AmBe neutron calibration as control measurements in side bands.
- The calibration data is used to measure the fraction of Gamma background and expected signal in a given band
- Uncertainties in the energy scale due to L_{eff} are also taken as side band measurements with a wide uncertainty in the low recoil energy range
- Astrophysical uncertainties are naturally built into the model.
  e.g. v_{esc} is taken as a measurement with v_{obs}=544 km/s (498km/s < vesc < 608km/s)
  (Smith et. al. arxiv:astro-ph/0611671)



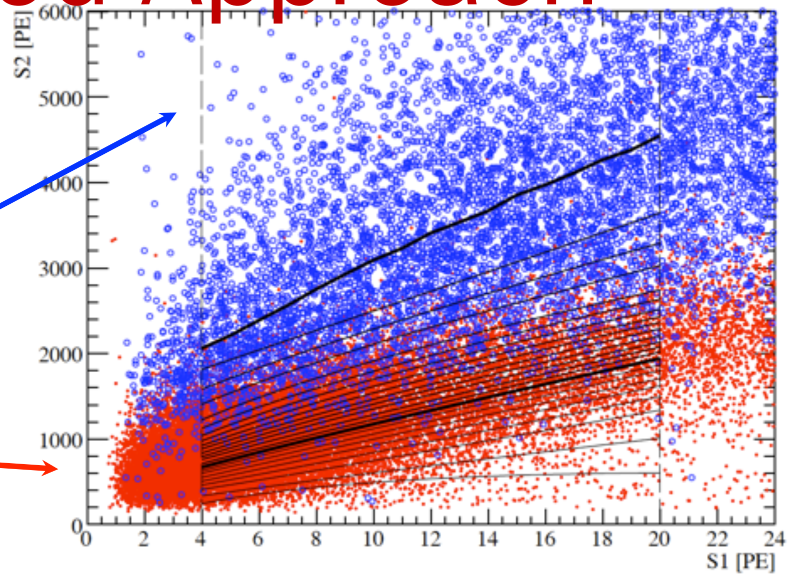


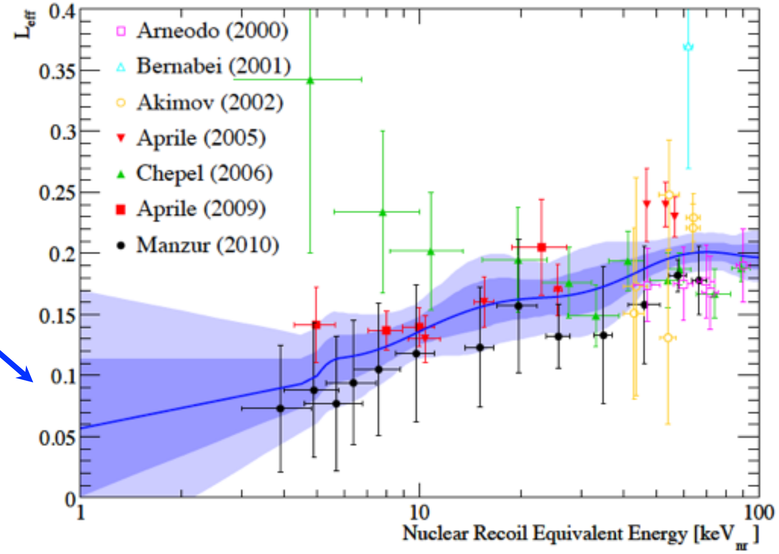
The Profile Likelihood Approach

σ   par of interest

unknown total BG

$$\mathscr{L} = \mathscr{L}_1(\sigma, N_b, \epsilon_s, \epsilon_b, \mathcal{L}_{\rm eff}, v_{\rm esc}; m_\chi) \times \mathscr{L}_2(\epsilon_s) \times \mathscr{L}_3(\epsilon_b) \times \mathscr{L}_4(\mathcal{L}_{\rm eff}) \times \mathscr{L}_5(v_{\rm esc}).$$





$\sigma(m_\chi)$ - WIMP cross section
ε_b - fraction of BG
ε_s - fraction of signal
v_{esc} - escape velocity
L_{eff} - energy calibration

The Profile Likelihood Approach

- The Profile Likelihood ratio profiles all nuisance parameters

$$\lambda(\sigma) = \frac{\max\limits_{\sigma \text{ fixed}} \mathscr{L}\left(\sigma; \mathcal{L}_{\text{eff}}, v_{\text{esc}}, N_b, \epsilon_s, \epsilon_b\right)}{\max \mathscr{L}\left(\sigma, \mathcal{L}_{\text{eff}}, v_{\text{esc}}, N_b, \epsilon_s, \epsilon_b\right)}$$

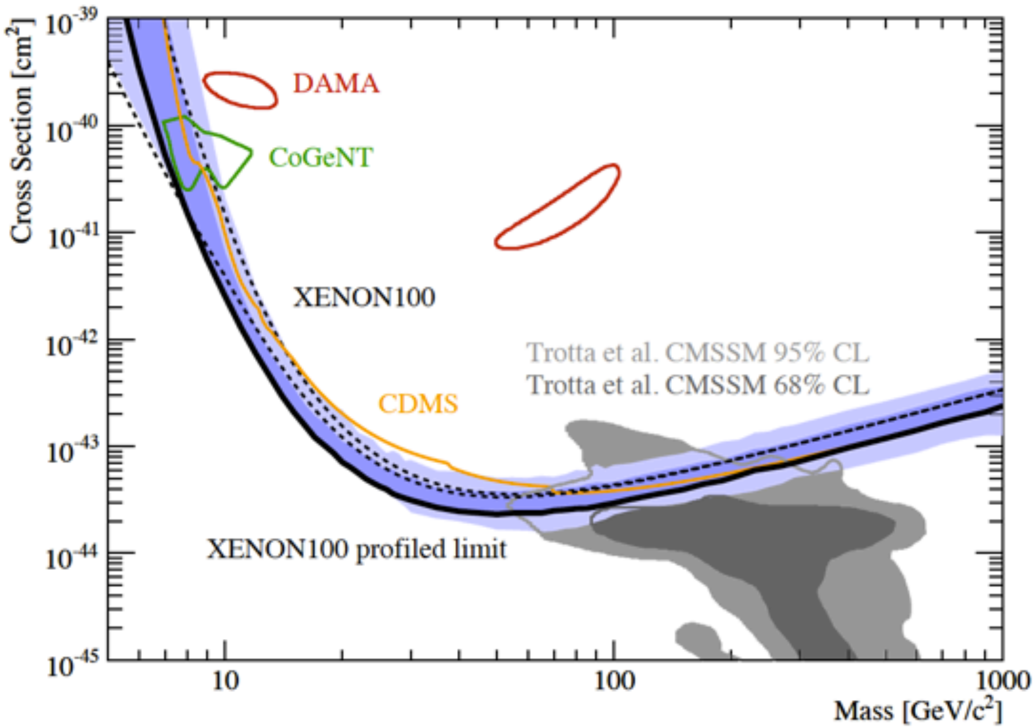
- The upper limit $\sigma^{up}(m_\chi)$ on the cross section σ for a given WIMP mass m_χ is found by testing the signal hypothesis and find the maximum cross section, σ, which rejects the signal hypothesis at the 90% Confidence Level

- i.e. find the $\sigma(m_\chi)$ such that the probability to reject the signal hypothesis when the signal is true <10%. Obviously very high cross sections are excluded in the absence of observed events.

The Profile Likelihood Approach

The XENON100 Collaboration
Likelihood Approach to the First Dark Matter Results from XENON100
arXiv:1103.0303v1 [hep-ex] 1 Mar 2011

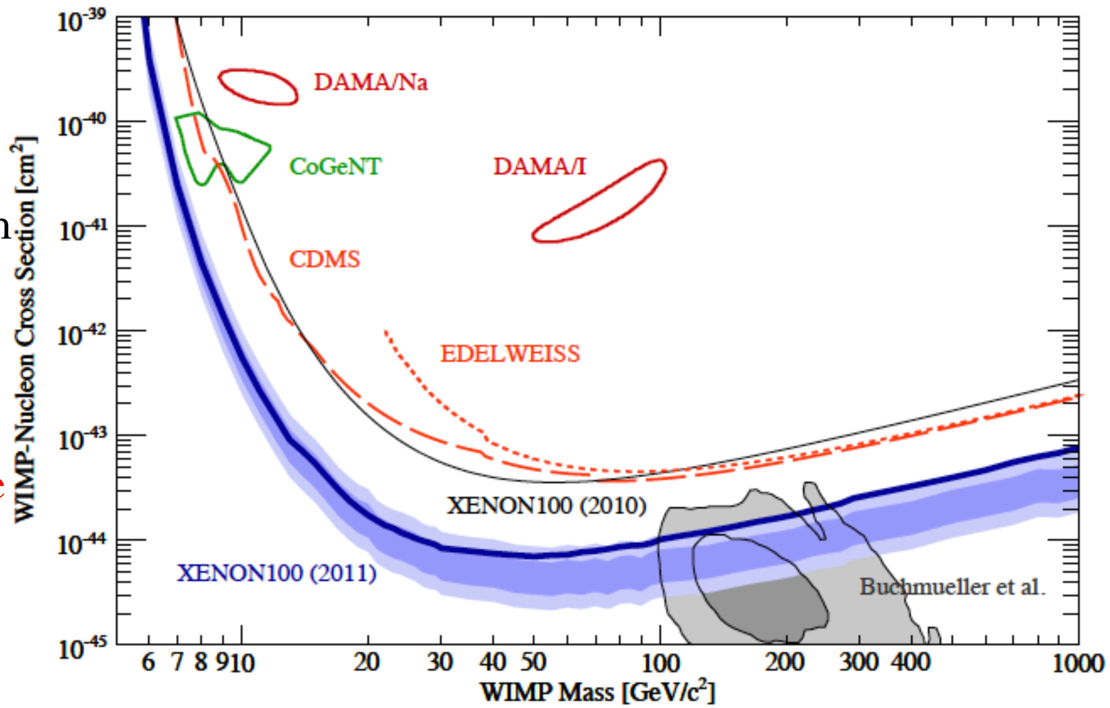
* Applying a likelihood ratio method to XENON100 first data, results in an improvement of the limit
* $\sigma^{up} < 2.4\times10^{-44}$ cm^2 for WIMPs with $m_\chi = 50$ GeV



The Profile Likelihood Approach

The XENON100 Collaboration
Dark Matter Results from 100 Live Days of XENON100 Data
arXiv:1104.2549v2 [astro-ph.CO] 29 Apr 2011

- Applying a likelihood ratio method to XENON100 first data, results in an improvement of the limit
- $\sigma^{up} <$ ~~2.4×10^{-44}~~ 7.0×10^{-45} cm^2 for WIMPs with m_χ=50 GeV
- The Profile Likelihood analysis has led to the stronger limit over a wide range of WIMP masses. CoGeNT results highly disfavored



Summary

- The profile likelihood is used successfully in both HEP (LHC) and Astroparticle (XENON).
- Asymptotic distributions exist for both the null and the alternate hypotheses and save lots of computing time
- Asimov data sets are used to estimate median discovery and exclusion sensitivities
- Lots of progress in related issues for LHC
  - Look Elsewhere Effect – 1D
  - CLs→PCL
- Look Elsewhere Effect is also applicable to search for signals in the sky (in its 2-D approximated form). Formulae were derived which might save enormous computing time.

BACKUP

Nuclear Recoil Scale (L_{eff})

- The WIMP interacts with Xe nucleus
  -->Nuclear recoil (NR) scintillation
- β and γs generate Electron Recoils
- Absolute measurement of NR scintillation yield is difficult, in particular at low recoil energies (below 5-6 keV_{nr})
- Measurement is done relative to ^{57}Co 122keV

$$L_{eff}(E_{nr}) = \frac{Ly(E_{nr})}{Ly(122KeV_{ee})}$$

- Leff is crucial, it sets the scale between S1 and the recoil energy

Signal (pe)

Quenching of scintillation yield for 122 KeV gammas due to field (0.58 at~0.5 KV/cm)

$$E_{nr} = \frac{S1}{Ly(122KeV_{ee})} \cdot \frac{S_{ee}}{S_{nr}} \cdot \frac{1}{L_{eff}}$$

Nuclear recoil energy

- Ly(122 KeV)=2.2±0.1 PE/keV_{ee}
  Field Correction S_{ee}=0.58
  S_{nr}=0.95

Light yield for 122 KeV in pe (~2.2 pe/KeV)

Quenching of scintillation yield for NRs due to field (0.95 at~0.5 KV/cm)

Relative scintillation efficiency of NRs to 122 KeV gammas at zero field