probabilistic description of stellar ensembles

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What I understand as an stellar ensemble?

- Any set of stars whatever their origin, physical connection, environment conditions etc. where i just can access their integrated light

- (no priors for the moment)

What I look for?

- Description of the ensemble as a global system and to obtain their relevant physical properties: age distribution of the stars in the ensemble, amount of mass into stars, metallicity distribution (i.e. describe the ensemble in terms of stellar populations)

Which kind of data I have to obtain that information?

- Just the resulting SED (spectral energy distribution) of the system, but incomplete (not all wavelengths, and variable resolution)

- Sometimes, several SEDs fractions (photometry points) in different areas (galaxy image, IFUs). [For these cases, some priors are useful]

What I want?

- I would like an accurate description of stellar ensembles, (and to increase the precision in analysis, if possible, after that)

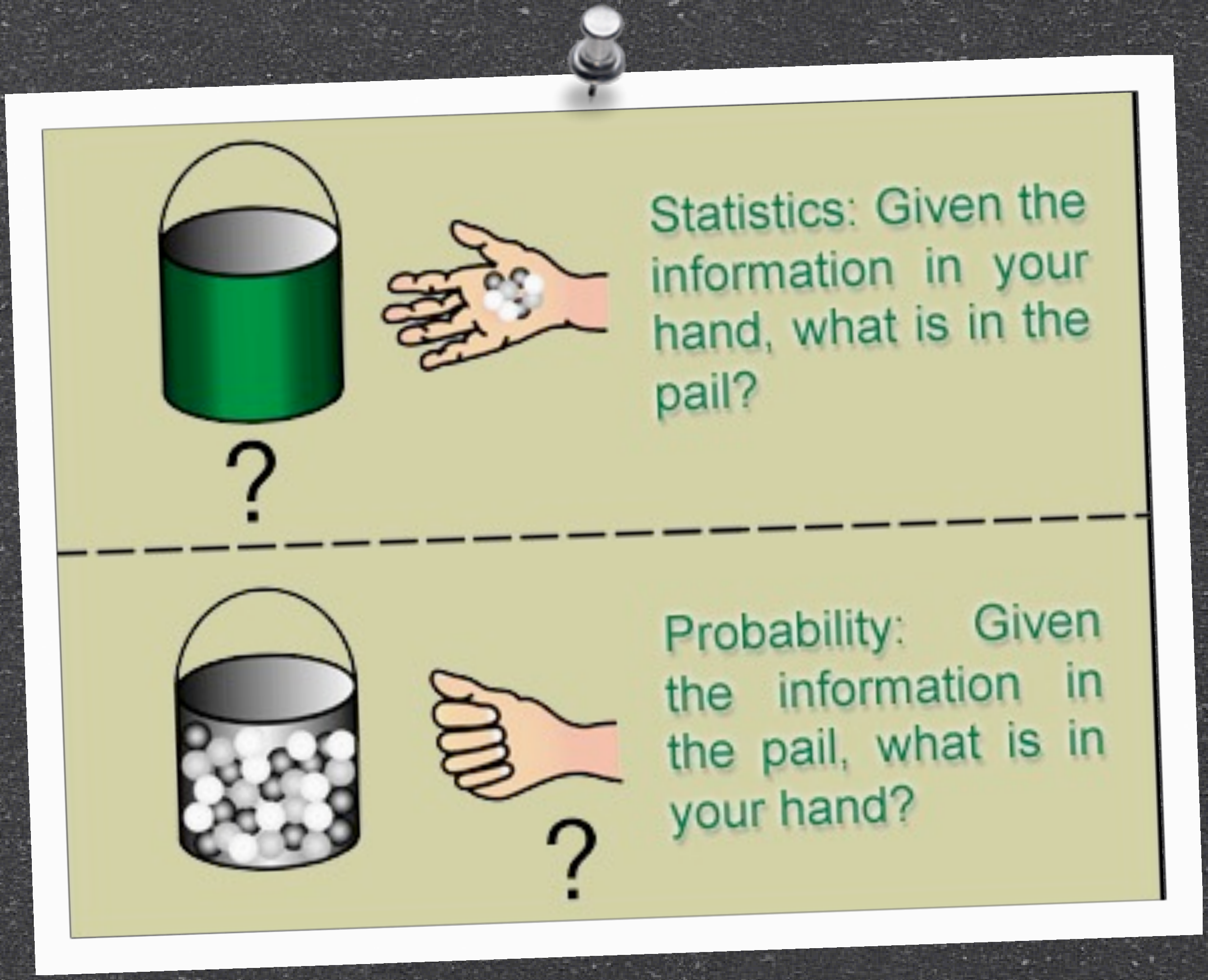
- Ok, I am in astro-statistics workshop, but most of time my problem is to deal with a single “data point” (the SED) and I do not want to talk about precision (nor about goodness of fit) but about accuracy

- So I have a problem...

- ... or maybe not...

Key point 1

statistics and probability



to do statistics (inference from data and model comparison) right, be must do probability (to know/evaluate the degree of confidence of the model results) before

Ok, probability of properties on stellar ensembles. How to proceeded?

- The relevant point of my problem are stars

- I know that the distribution of properties of stars in an ensemble according the ensemble physical conditions is the responsible of the integrated light

- So let see how the properties of stars are distributed..

CMD diagram from Hipparcos

We see, at least,
two defined areas

-The Post Main Sequence:

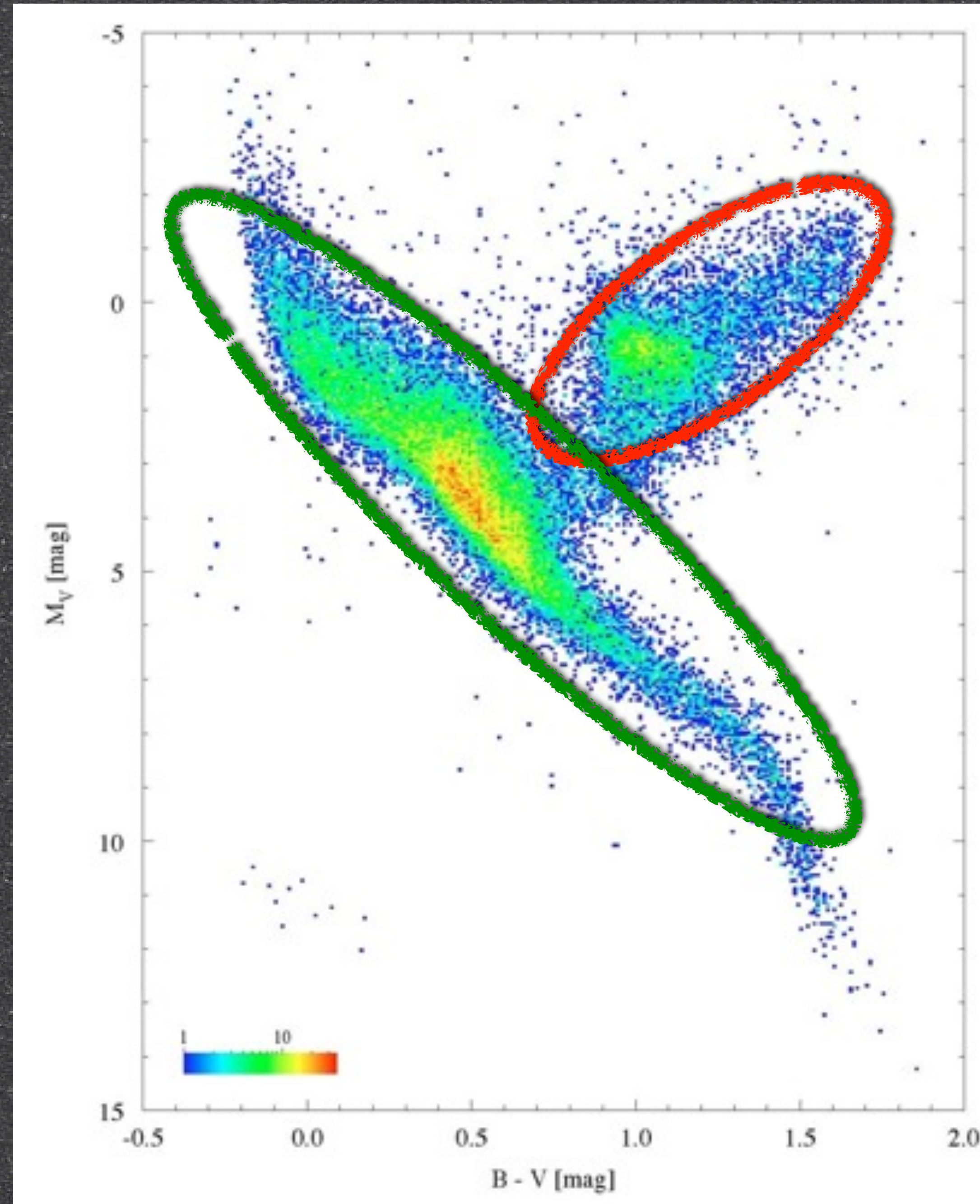
with an stellar density proportional to the life of different evolutionary phases given by stellar evolution

-The Main Sequence:

with an stellar variable stellar density:

- Low density of low luminosity stars (incompleteness)
- Stepper density of high luminosity stars than expected by MS life time:

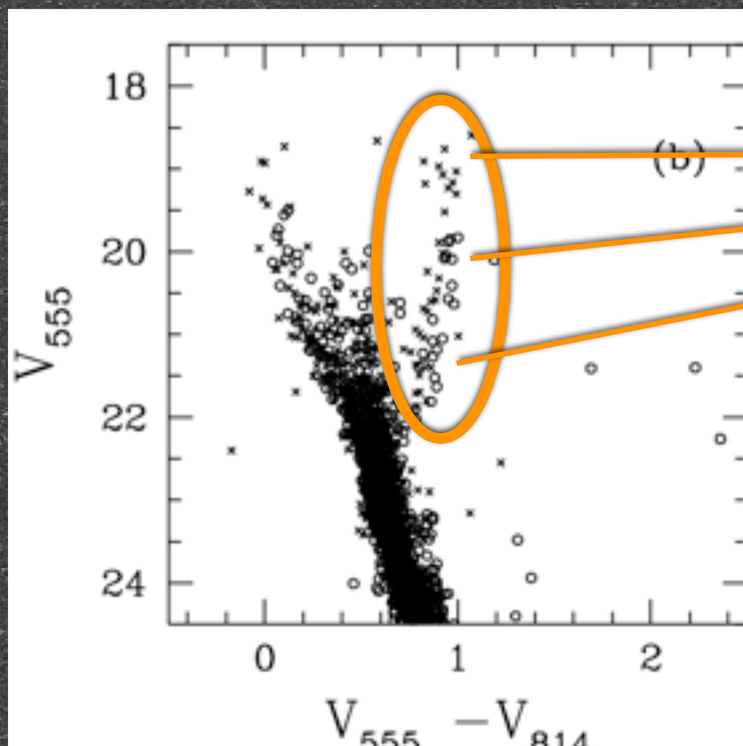
Not all stars born with the same probability: More massive stars are intrinsically less frequent (the IMF)

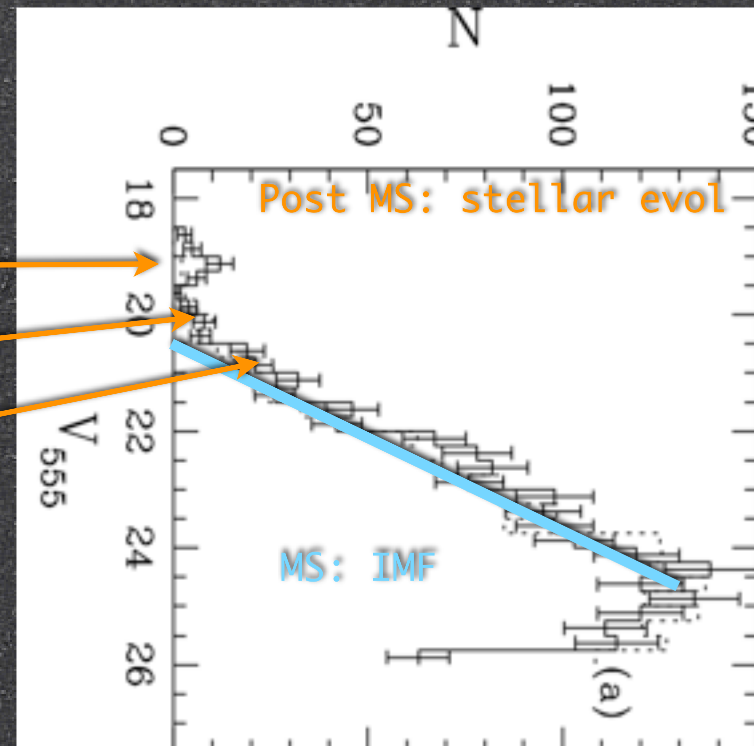


And it is common in all CMD diagram you can see

It allows to model the stellar Luminosity (distribution) function, sLDF, of an ensemble

Fig from Javiel, Santiago,Kerber A&A 431, 73 (2005)





IMF: Initial mass function

CMD fit is equivalent to characterize the sLDF for different bands

Let's see properties of the sLDF

Let us assume a system where all stars are in the MS and that the stars follows a mass-luminosity relation:

$$\ell \propto m^{\beta}$$

Assuming a power-law IMF, $\phi(m) \propto m^{-\alpha}$, we can define the sLDF as

$$\varphi_{\mathrm{L}}(\ell) = \phi(m) \times \left(\frac{d\ell(m)}{dm}\right)^{-1} = A\,\ell^{-\frac{\alpha}{\beta}} \cdot \frac{1}{\beta}\,\ell^{-\frac{\beta-1}{\beta}} = \frac{A}{\beta}\,\ell^{\frac{1-\alpha-\beta}{\beta}}.$$

the IMF expressed in ℓ instead m **X** The factor due to the variable change

Let's see properties of the sLDF

The mean value of the sLDF is then:

$$\mu'_1 = \frac{A}{\beta}\int_{\ell_{\min}}^{\ell_{\max}} \ell \cdot \ell^{\frac{1-\alpha-\beta}{\beta}} d\ell = \frac{A}{1+\beta-\alpha} \cdot \left(\ell_{\max}^{\frac{1+\beta-\alpha}{\beta}} - \ell_{\min}^{\frac{1+\beta-\alpha}{\beta}}\right)$$

If $1+\beta-\alpha > 0$, the mean luminosity is driven by $\ell_{\max}$. In a typical situation with $\beta \approx 3$, the most luminous stars will dominate the luminosity if $\alpha < 4$: this is the case of Salpeter's IMF.

sLDF as a wild distribution

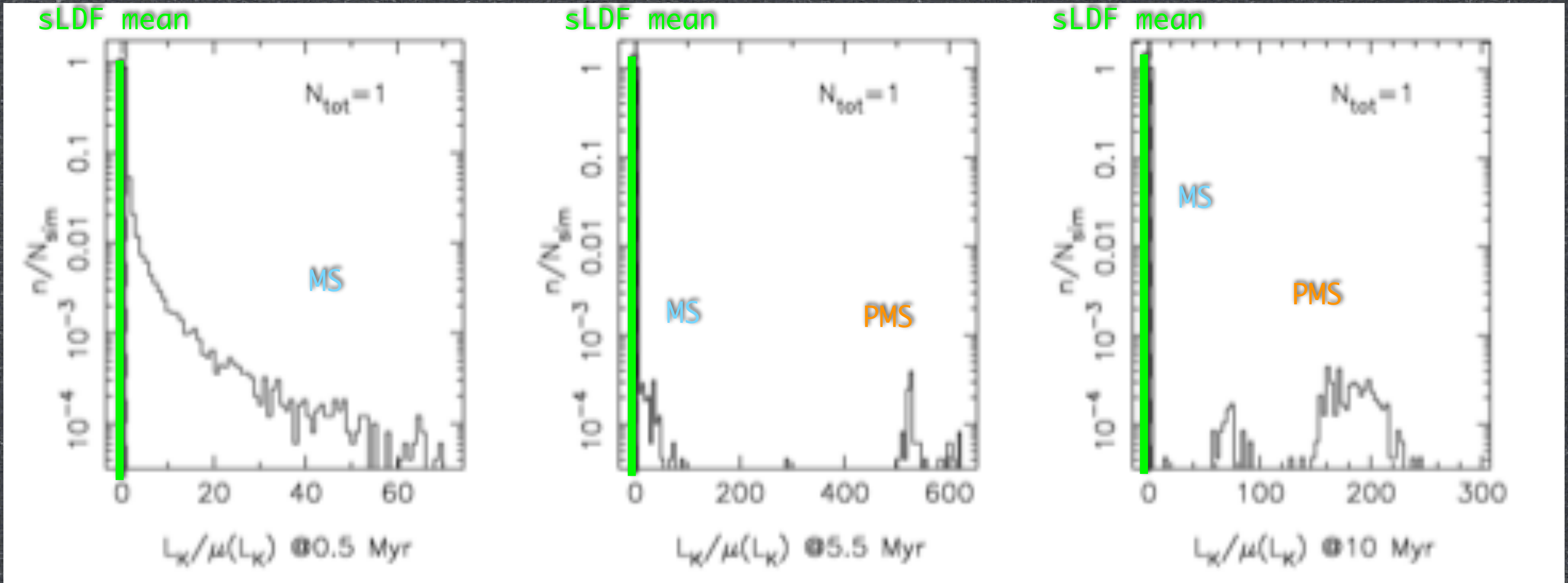
A wild distribution is one where the extreme value dominates, although with low probability, dominates the mean

We just see that it is the case of the sLDF. So the “integrated” mean light is dominated by the most luminous stars (see later)

It has an advantage:

the most luminous stars are the Post-MS ones, that gives most the information about the age of the system (we have solve half of our problem!).

The sLDF

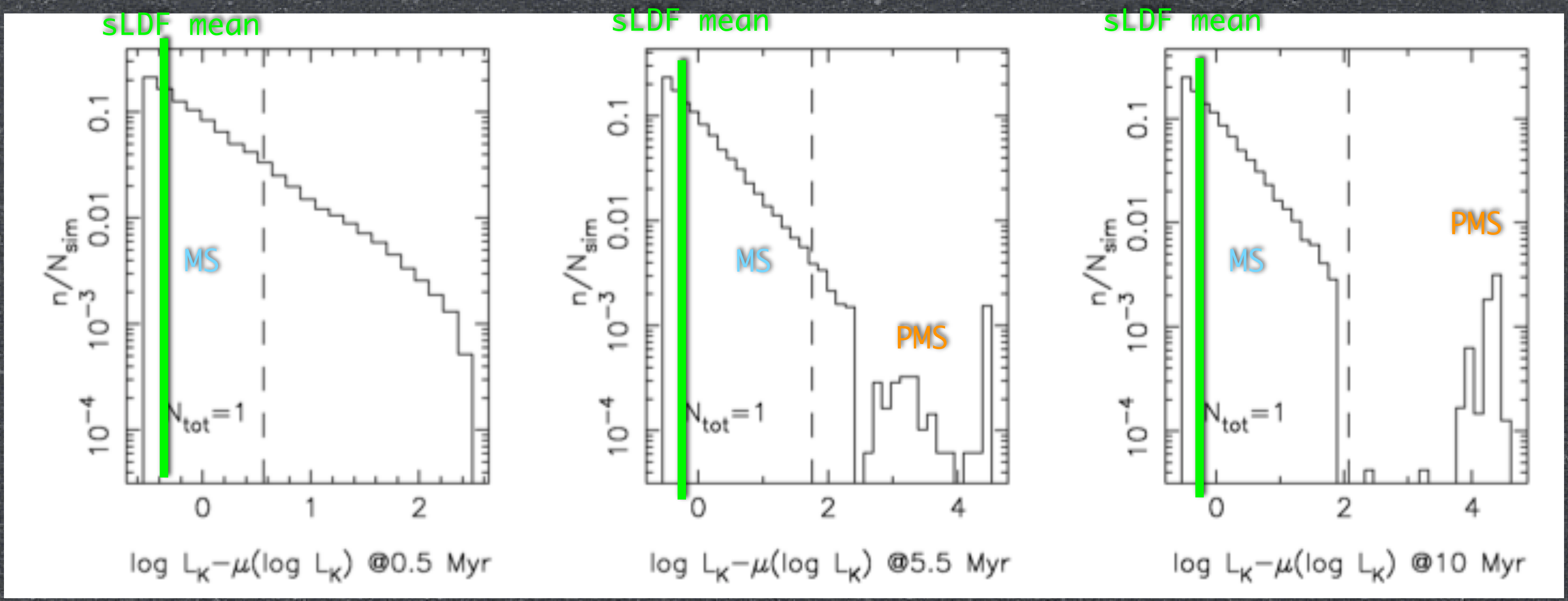


Cerviño & Vals-Gabaud MNRAS 388, 481 (2003)

Montecarlo simulation of the sLDF in L(K) for 0.5, 5.5 and 10 Ma 1 star “clusters”

Note: - It looks that the mean is not always a “good” characterization

The sLDF



Caution: A better sLDF visualization does not implies a better sLDF characterization

sLDF as a wild distribution

A wild distribution is one where the extreme value dominates, although with low probability, dominates the mean

We just see that it is the case of the sLDF. So the “integrated” mean light is dominated by the most luminous stars (see later)

It has an advantage:

the most luminous stars are the Post-MS ones, that gives most the information about the age of the system (we have solve half of our problem!).

And a problem:

we can infer information about “unseen” stars just in case de IMF is “well sampled”, i.e. the number of stars in the system is large enough (N > 1e5 stars in V or N > 1e7 stars in IR)

Ok, Lets back to stellar ensembles. How to proceeded?

- The stellar luminosities are distributed following a "wild" (theoretical) sLDF

- It allows obtain physical parameters of the ensemble like ages, number of stars etc, ensemble mass...

- ... as far as our ensemble is a representative of the theoretical distribution (enough number of stars), but the number of stars is unknown!

- So let see how the properties of ensembles are distributed..

sLDF and stellar ensembles (1/3)

$\varphi_1(\ell) = \text{Stellar LDF (1 star)}$
$\varphi_{\rm N}(L) = \text{Population LDF of N stars}$

1 star: $$\varphi_1(L) = \varphi_1(\ell)$$

2 stars: $$\varphi_2(L) = \int \varphi_1(\ell)\,\varphi_1(L-\ell)\,d\ell = \varphi_1(\ell) \otimes \varphi_1(\ell)$$

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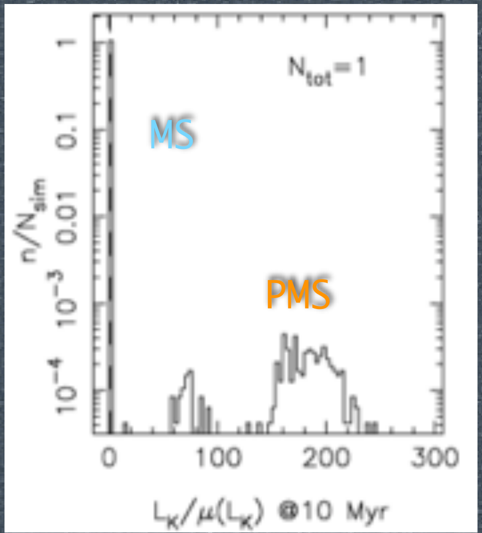
N stars: $$\varphi_{\rm N}(L) = \overbrace{\varphi_1(\ell) \otimes \varphi_1(\ell) \otimes \ldots \otimes \varphi_1(\ell)}^{N \text{ times}}$$

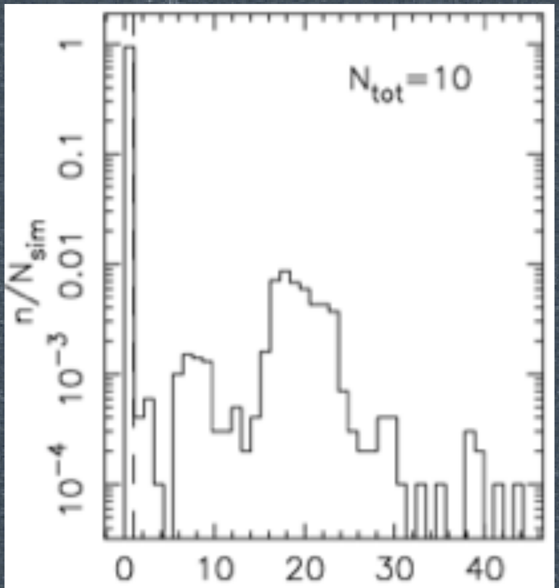
More details in Cerviño & Luridiana 2006 A&A 451, 475
(see also Selman & Melnick 2008 ApJ 689, 816)

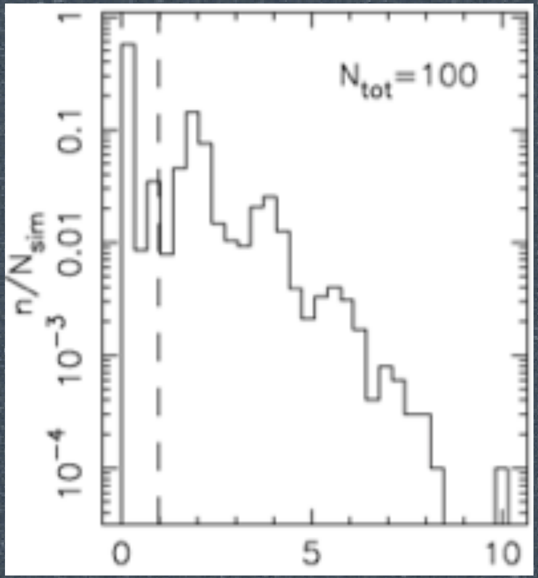
sLDF and stellar ensembles (2/3)

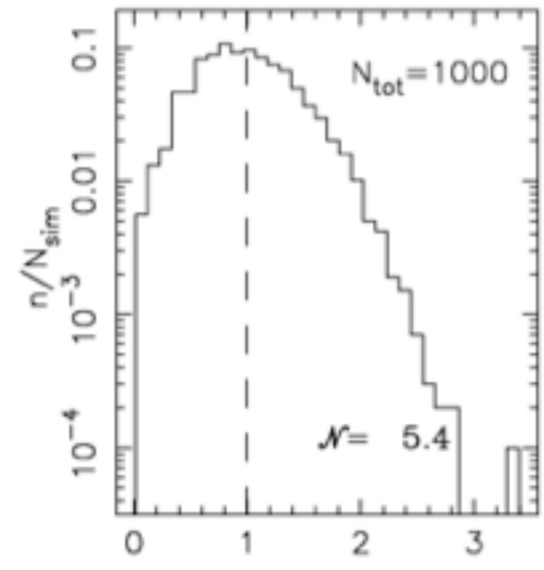


Single star

Clusters







The sLDF shape provides the “integrated” LDF shape

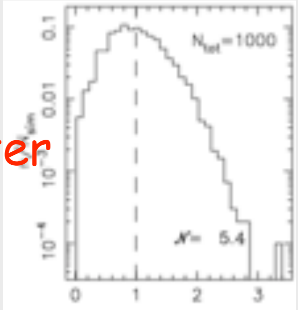
Ntot from Slapeter IMF in the 2-120 Mo range

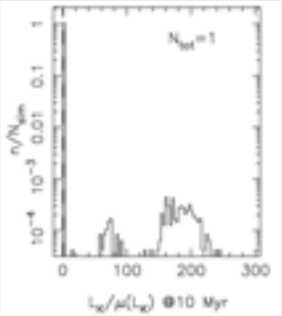
sLDF and stellar ensembles (3/3)

The sLDF mean is useful so far...

$\langle L \rangle = N \times \langle \ell \rangle$	mean
$\sigma^2(L) = N \times \sigma^2(\ell)$	variance
$\Gamma_1(L) = \frac{1}{\sqrt{N}} \times \gamma_1(\ell)$	skewness
$\Gamma_2(L) = \frac{1}{N} \times \gamma_2(\ell)$	kurtosis

Cluster





stellar

Actually, synthesis models results is just the sLDF characterization!

$$\langle \ell_\lambda \rangle = \mu'_1(\ell_\lambda)$$

The mean of the sLDF is the (only) result of classical synthesis models (SB99, B&C, etc)

$$\frac{1}{N_{\rm eff}} = \frac{{\rm var}(\ell_\lambda)^{1/2}}{\langle \ell_\lambda \rangle}$$

$$= \frac{\kappa_2(\ell_\lambda)^{1/2}}{\mu'_1(\ell_\lambda)}$$

But the variance shows that not all wavelength points are equivalent!!!

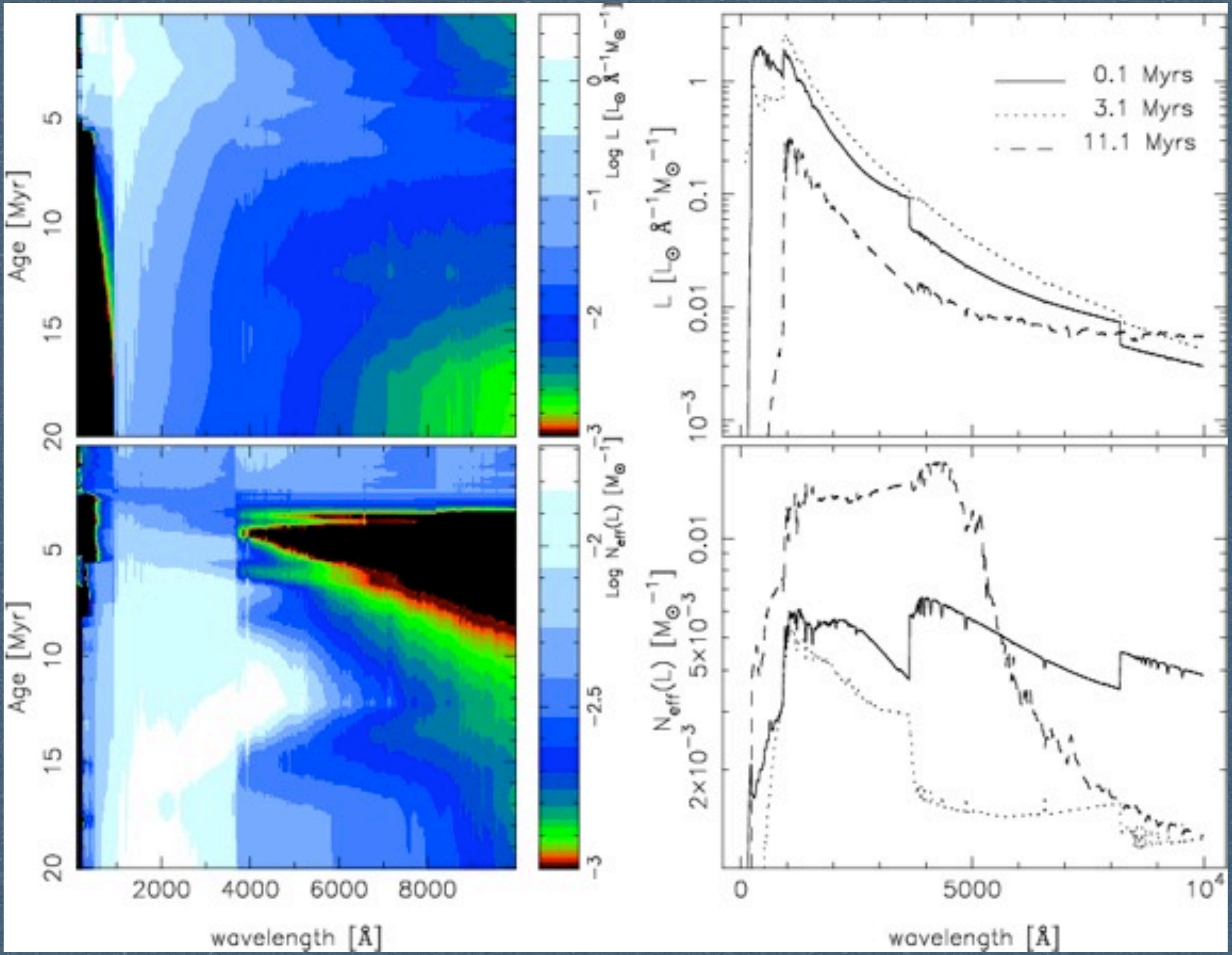
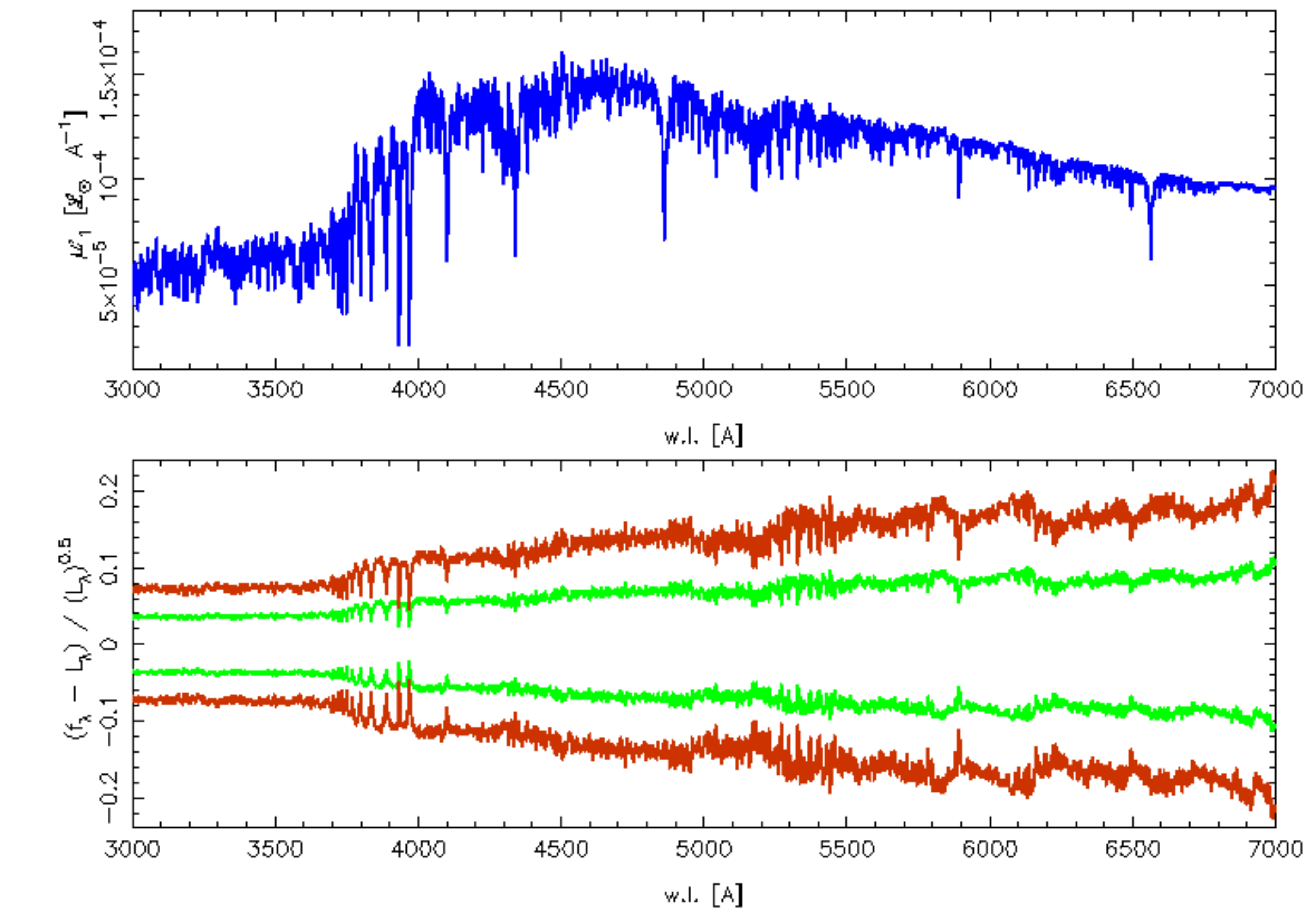


Fig form Cerviño et al. (2002 A&A 381, 51)

If we have enough stars in our system (which is a decision of Nature), the pLDF of the SED are a set of gaussians for each wavelength.

And we can define a measure of the dispersion independent on the number of stars in the system (also called Surface Brightness Fluctuation, SBF)

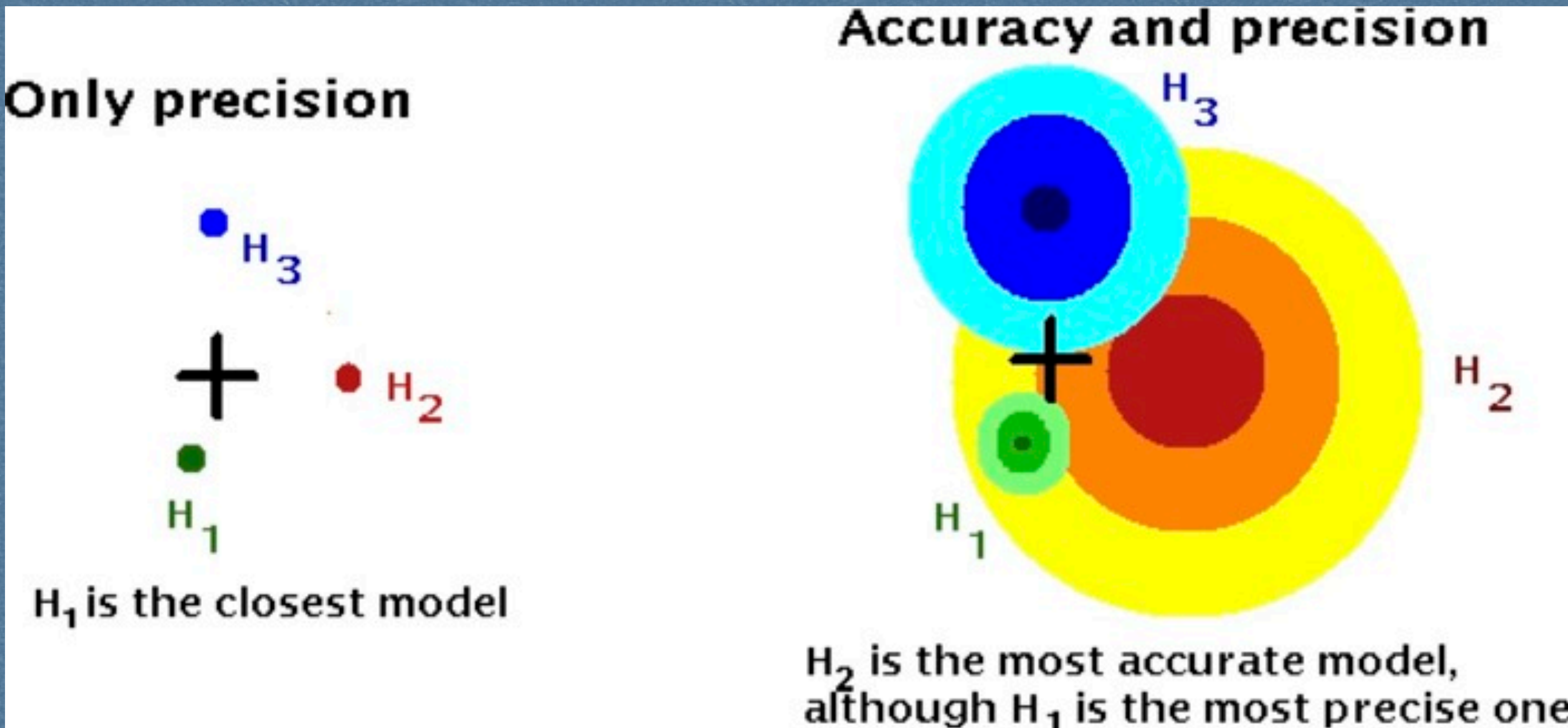
$$\bar{\mathcal{L}} = \frac{\mathrm{var}(\mathcal{L})}{\langle \mathcal{L} \rangle} = \frac{\mathrm{var}(\ell)}{\langle \ell \rangle} = \bar{\ell}$$



3000
3500
4000
4500
5000
5500
6000
6500
7000
w.l. [A]
w.l. [A]
5×10⁻⁵
10⁻⁴
1.5×10⁻⁴
−0.2
−0.1
0
0.1
0.2

Key point 2

So each stellar ensemble physical conditions defines its own "metric of fitting"



Conclusions

- An stellar ensemble must be described as a probability distribution for the given ensemble physical conditions (since it is the result of a distributed mixture of stars)

- Not only mean values must be used, but also the other moments of the distribution, which defines a theoretical metric of fit

- Increasing spatial resolution implies a larger intrinsic variance (low number of stars per resolution element)...

.... but, several pieces of high resolution elements (e.j. IFUs) allows to sampling better the underlying pLDF (and to apply agregation, segregation or cross-matching techniques)

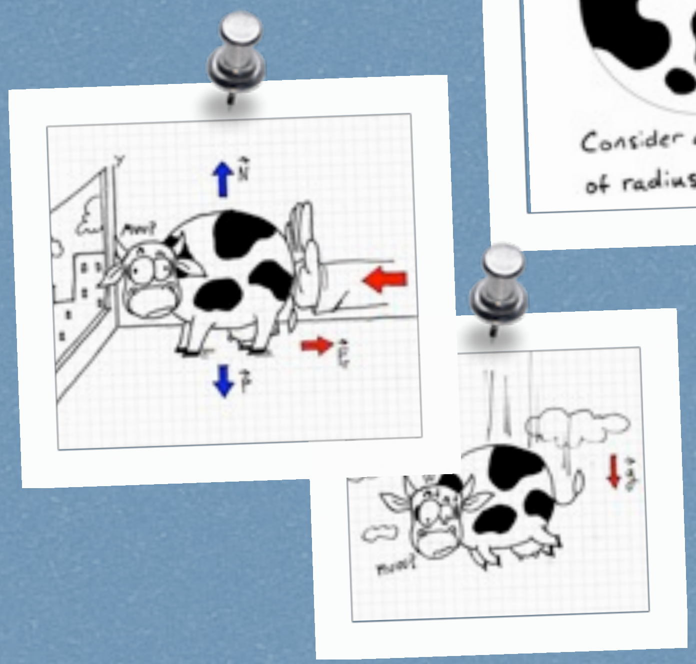
Conclusions

- The residual of the fit contains valuable physical information!
(and any smoothing technique/presentation can erase this information)

- To be done:
    - How correlates different wavelengths/bands each other?
      (i.e. what is the effective amount of information we can obtain from the SEDs?)
    - How work for system outside the gaussian regimen? (e.j. small clusters)
      (pLDF becomes difficult to handle, appearing bimodal and multimodal pLDF; Monte Carlo? Data Mining over them?)

Two final notes:

- You can not be more precise than your model!



Note: A spherical cow is not a precise model, but it can be a accurate one

Precision can be mathematically defined always

Accuracy is context dependent!
(physics and data is here)

Two final notes:

- Psycological bias

Human mind is designed to survive by quick detection of out-layers (sort-term tension) and steady-state recognition of regular patterns (long-term relax)

But human beings do not like too much intermediate situations (like high dispersion) neither they are designed to work with probability (neither probability distributions) where intuition just fails

Human beings tend to prefer a precise smooth values rather than accurate distribution of results

but a smoothed results can erase valuable physical information